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THEORIA MOTUUM PLANETARUM ET COMETARUM.

CONTINENS

METHODUM FACILEM EX ALIQUOT OBSERVATIONIBUS ORBITAS CUM PLANETARUM TUM COMETARUM DETERMINANDI.

UNA CUM CALCULO, QUO COMETÆ, QUI ANNIS 1680. ET 1681. ITEMQUE EJUS, QUI NUPER EST VISUS, MOTUS VERUS INVESTIGATUR.

AUCTORE LEONHARDO EULERO.

Literary Society of Bombay
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De Motu planetarum & cometarum circa ſolem motorum.

I.

Si leges motus, quas planetæ primarii atque cometæ in curſu ſuo circa ſolem obſervant, conſulamus, non ſolum ellipſes, ſed etiam omnis generis ſectiones conicæ orbitas cometarum repræſentare poſſunt. Neque enim aliud diſcrimen inter planetas & cometas intercedit, niſi orbitæ figura, quæ ſi fuerit ellipſis non multum a circulo recedens, ſidus in ea motum planetæ nomen obtinuit, ſin autem orbita vehementer a circulo abhorreat, ſive ſit ellipſis admodum oblonga, ſive parabola, ſive adeo hyperbola, ejusmodi ſidera cometæ appellantur. Utrique autem, planetæ ſcilicet & cometæ, in curſu ſuo easdem ſequi leges motus merito ſtatuuntur, ſecundum quas tempora, quibus data ſpatia in orbitis ſuis abſolvunt, ſint in ratione compoſita arearum circum ſolem deſcriptarum directa, & reciproca ſubdu-

 plicata

plicata laterum rectorum, ſeu ut vocari alias ſolent, parametrorum orbitarum: Sol autem in altero foco cujusque orbitæ fixus æſtimatur. Ex hac autem lege non ſolum planetarum cometarumve motus in orbibus ſive ellipticis ſive parabolicis ſive hyperbolicis definiri, ſed etiam hæ ipſæ orbitæ per obſervationes aliquot determinari poſſunt.

2. Ex iſto etiam fonte derivatæ ſunt tabulæ aſtronomicæ motuum planetarum, ope quarum ex dato tempore ſeu anomalia media inveniri ſolet anomalia vera ſeu coæquata, & viciſſim; quæ methodus vero tantum ad orbitas ellipticas circulo vicinas eſt accommodata. Quare ſi eandem theoriam in ſenſu latiſſimo ſtabilire atque ad motus etiam cometarum, qui in hyperbolis incedant, transferre velimus, ab inſtituto hoc conſueto, quo relatio inter anomaliam veram & mediam definiri ſolet, recedere, alioque modo curſum ad calculum revocare oportebit. Primum enim incommodum methodi receptæ in hoc conſiſtit, quod anomaliæ ab aphelio computentur: aphelium autem ſolæ orbitæ ellipticæ admittunt, parabolicæ autem & hyperbolicæ eo penitus carent. Quamobrem huic incommodo medela afferetur, anomalias a perihelio, quod in omnis generis orbitas æque competit, computando. Deinde etiam anomalia media uti non licebit, quod a tempore periodico pendeat, orbitæ autem parabolicæ & hyperbolicæ tempore periodico deſtituantur. Loco anomaliæ mediæ igitur ipſum tempus a momento, quo ſidus in perihelio verſatur, computatum adhibebo; anomalia vera autem erit diſtantia ſideris heliocentrica a perihelio. His

igitur

igitur præmiſſis ſequentia problemata evolvam, ex quibus natura motuum coeleſtium cum theoretice cognosci, tum etiam uſus in praxi dilucide perſpici poterit.

Problema I.

3. Data area, quam planeta vel cometa dato tempore circa ſolem deſcripſit, invenire latus rectum orbitæ.

Solutio.

Quoniam hic quantitates diverſi generis, areæ ſcilicet & tempora inter ſe conferuntur, utrumque genus ſecundum certam conſtantemque menſuram exprimi debebit. Poſita igitur diſtantia Solis a terra media $=$ 100000, in ejusmodi unitatibus cunctas magnitudines exprimemus; atque adeo latus rectum orbitæ, quod quærimus, ex hac unitate definiri oportet. Aream autem circa ſolem deſcriptam per eandem menſuram in ejusmodi partibus quadratis, quarum 100000 ſemiaxem orbis magni conſtituunt, exhiberi pono. Sit igitur A area, quam planeta cometave circa ſolem deſcripſit, ad menſuram aſſumtam revocata; atque b denotet ſemiſſem lateris recti, ſeu applicatam ad axem transverſum in foco normalem, in eadem menſura definiendam. Tempus denique poſthac perpetuo exprimamus in diebus naturalibus temporis medii, & fractionibus diei decimalibus: ſitque tempus propoſitum, quo area A circa ſolem confecta perhibetur, $=$ T diebus; hoc igitur modo quantitates A, b, & T, quarum in hoc problemate mentio fit, ad numeros abſolutos reducentur. Cum igitur ex natura motus corporum coeleſtium circa ſolem motorum ſit tempus

T proportionale areæ A diviſæ per $\sqrt{b}$, fiet $\frac{A}{T\sqrt{b}}$ = numero cuidam conſtanti, qui ſit = m; ita ut ſit $T = \frac{A}{m\sqrt{b}}$ ſeu $\sqrt{b} = \frac{A}{mT}$, & $b = \frac{AA}{mmTT}$; dummodo ergo iſte numerus m fuerit cognitus, problema erit reſolutum, eo quod ſemilatus rectum b reperitur expreſſum in ejuſmodi partibus, quarum 100000 diſtantiam ſolis a terra mediam conficiunt. Ad numerum itaque m definiendum caſum jam cognitum evolvamus. Cum ſcilicet conſtet, terram circa ſolem in orbita ſua circumferri tempore anni ſiderei ſeu $365^d, 6^h, 8', 30''$; ſi ponamus T = 365, 256, & A = areæ totius orbitæ terreſtris & b = ſemilateri recto orbitæ terræ, dabit fractio $\frac{A}{T\sqrt{b}}$ verum valorem numeri m. Sit ergo ſemiaxis tranſverſus orbitæ terræ = c, quem ſumſimus = 100000, erit ſemiaxis conjugatus = $\sqrt{bc}$. Denotet 1 : π rationem diametri ad peripheriam, ita ut ſit π = 3, 14159265; erit area circuli radio c deſcripti = πcc, quæ erit ad aream orbitæ terræ ut c ad $\sqrt{bc}$, unde fit area orbitæ terreſtris $A = \pi c \sqrt{bc}$, ideoque $m = \frac{A}{T\sqrt{b}} = \frac{\pi c\sqrt{c}}{T}$ ſeu numerus m dabitur per numeros cognitos π = 3, 14159265; c = 100000, & T = 365, 256. Hinc per logarithmos valor numeri m reperietur:

$l\,\pi$	=	0, 4971498727
$lc\sqrt{c}$	=	7, 5000000000
		7, 9971498727
ſubtr. lT	=	2, 5625973588
erit $l\,m$	=	5, 4345525139
ideoque m	=	271989, 735

Inven-



J. H. Friſch ſc. Berol.

Invento ergo numero $m = 271989,735$, erit orbitæ cujuscunque circa ſolem deſcriptæ latus rectum $2b = \frac{2AA}{mmTT}$. Q. E. J.

Coroll. 1.

4. Si igitur cognita fuerit area, quam cometa dato tempore circa ſolem conficit, hinc latus rectum orbitæ, in qua movetur cometa, reperitur ita, ut ejus ratio ad diſtantiam Solis a terra mediam exhibeatur.

Coroll. 2.

5. Sin autem latus rectum orbitæ, quod ſumimus $= 2b$ jam fuerit cognitum, tempus aſſignari poterit, quo data orbitæ area circa ſolem abſolvitur. Sit enim area hæc deſcripta $= A$ erit tempus $T = \frac{A}{m\sqrt{b}}$ dier: ſiquidem, quod ubique eſt tenendum, longitudines in ejusmodi partibus exprimantur, quarum 100000 ſemiaxem transverſum orbis magni conſtituunt, & ſit $m = 271989,735$.

Coroll. 3.

6. Viciſſim ergo ex dato latere recto orbitæ $2b$ area aſſignari poteſt, quam planeta vel cometa dato tempore T circa ſolem deſcribit: tempore enim T in diebus expreſſo erit area deſcripta $A = mT\sqrt{b}$.

Problema II.

7. Datis ſectionis conicæ ABM diſtantia verticis a foco AS una cum latere recto, cujus ſemiſſis eſt BS, invenire Fig. 1.

 rela-

relationem, quæ inter diſtantiam cujusvis puncti M a foco S & anomaliam veram ſeu angulum ASM intercedit.

Solutio.

Sit diſtantia verticis a foco AS, ſeu pro orbitis planetarum cometarumve diſtantia perihelii A à ſole S nempe $AS = a$, & ſemiſſis lateris recti, nempe applicata $BS = b$. Dabit ergo recta AS producta axem ſectionis conicæ, ad quem ex puncto M demittatur perpendiculum MP; erit ex natura ſectionum conicarum diſtantia $MS = a + \frac{(b-a)}{a}.AP$. Cum autem ſit $AP = a + PS$; ſi ponatur diſtantia $MS = y$ & angulus ſeu anomalia vera $ASM = v$, erit poſito ſinu toto $= 1$, $\operatorname{coſ} v = \frac{-PS}{MS} = \frac{-PS}{y}$, ideoque $PS = -y \operatorname{coſ} v$, & $AP = a - y \operatorname{coſ} v$. Hoc ergo valore ſubſtituto fiet $MS = y = a + \frac{(b-a)}{a}(a - y \operatorname{coſ} v) = b - \frac{(b-a) y \operatorname{coſ} v}{a}$. Hinc igitur prodit $\operatorname{coſ} v = \frac{a(b-y)}{y(b-a)}$ & $y = \frac{a \cdot b}{a + (b-a) \operatorname{coſ} v}$. Ex data ergo anomalia vera v reperitur diſtantia planetæ ſeu cometæ a ſole y; & viciſſim ex cognita hac diſtantia definitur anomalia vera. Q. E. J.

Coroll. 1.

8. Si anomalia vera ASM evaneſcit, fitque $v = 0$, erit $\operatorname{coſ} v = 1$. hocque caſu prodit diſtantia a ſole $y = a$; nempe puncto M in A incidente fit MS $(y) =$ AS (a). Simili modo ſi anomalia vera $ASM = v$ fiat angulus rectus, erit

coſ v

$\cos v = o$, & fiet $y = b$; id quod perſpicuum eſt, quoniam punctum M in B incidit.

Coroll. 2.

9. Quod ſi ponamus $v = 180°$, exhibebit y diſtantiam aphelii a ſole; erit autem ob $\cos v = -1$, hæc diſtantia $= \frac{ab}{2a-b}$; ad quam ſi addatur diſtantia perihelii a ſole $= a$ prodibit axis transverſus orbitæ $= \frac{2aa}{2a-b}$; & ſemiaxis transverſus $= \frac{aa}{2a-b}$, unde diſtantia foci a centro orbitæ erit $= \frac{a(b-a)}{2a-b}$; & excentricitas $= \frac{b-a}{a}$.

Coroll. 3.

10. Si igitur fuerit $b = a$, ſectio conica erit circulus, centrum in S & radium AS $= a$ habens. Sin autem ſit $b > a$, curva erit ellipſis, quoad b fiat $= 2a$, quo caſu curva abit in parabolam ob axem transverſum $\frac{2aa}{2a-b}$ infinitum. At ſi fiat $b > 2a$, tum axis transverſus fit negativus, quo indicatur, curvam eſſe hyperbolam.

Coroll. 4.

11. Si fuerit $b < a$, curva ſemper erit ellipſis, ſed punctum A erit vertex a foco S remotior, ideoque aphelium repræſentabit; punctum ergo orbitæ e diametro oppoſitum

 erit

erit perihelium, cujus a foco S diſtantia erit $= \frac{ab}{2a-b}$, quæ minor eſt quam a ſi fuerit $b < a$.

Problema III.

Fig. 2. 12. Si dentur duæ cometæ planetæve a ſole diſtantiæ FS & GS una cum angulo FSG intercepto; atque inſuper cognitum ſit latus rectum orbitæ; determinare ipſam orbitam AFG.

Solutio.

Sit ſemilatus rectum $= b$, ac ponatur diſtantia FS $= f$ GS $= g$, atque angulus FSG $= \Phi$; quæ ſunt data: ex quæſitis autem ſit diſtantia perihelii a ſole AS $= a$, & anomalia ſeu angulus ASF $= v$; erit angulus ASG $= v + \Phi$. His poſitis problema præcedens duas nobis ſuppeditabit æquationes; SF $= f = \frac{ab}{a + (b-a) \operatorname{cof} v}$ & SG $= g = \frac{ab}{a + (b-a) \operatorname{cof}(v+\Phi)}$, ex quibus pro a duplex oritur valor $a = \frac{bf \operatorname{cof} v}{b - f + f \operatorname{cof} v} = \frac{bg \operatorname{cof}(v+\Phi)}{b - g + g \operatorname{cof}(v+\Phi)}$; qui inter ſe æquati dant $(b-g) f \operatorname{cof} v = (b-f) g \operatorname{cof}(v+\Phi) = (b-f) g (\operatorname{cof} v . \operatorname{cof} \Phi - \operatorname{fin} v . \operatorname{fin} \Phi)$, ideoque $\frac{(b-g)f}{(b-f)g} = \operatorname{cof} \Phi - \operatorname{tang} v . \operatorname{fin} \Phi$, ex qua æquatione determinabitur anomalia v per quantitates mere cognitas, ſcilicet tang $v = \cot \Phi -$

$(b-g)$

$\frac{(b-g)f}{(b-f)g\,\mathrm{ſin}\,\phi}$ ſeu tang $v = \frac{(b-f)g\,\mathrm{coſ}\,\phi-(b-g)f}{(b-f)g\,\mathrm{ſin}\,\phi}$. Invento autem angulo ASF $= v$, diſtantia perihelii a ſole erit AS $= a = \frac{bf\,\mathrm{coſ}\,v}{b-f+f\,\mathrm{coſ}\,v}$. Ex datis vero AS $= a$ & ſemilatere recto $= b$ orbita AFG definitur per probl. præc. Q. E. J.

Coroll. 1.

13. Si igitur detur area FSG una cum tempore, quo planeta ſeu cometa ſpatium FG percurrit, per problema primum reperitur latus rectum, ac proinde ex hoc problemate definietur ipſa orbita AFG.

Coroll. 2.

14. Primo ſcilicet ex datis ſemilatere recto b & diſtantiis FS $= f$, GS $= g$ una cum angulo FSG $= \phi$ cognoscetur poſitio axis AS ex angulo ASF $= v$, cum ſit tang $v = \mathrm{cot}\,\phi - \frac{(b-g)f}{(b-f)g\,\mathrm{ſin}\,\phi}$; & cognito angulo v erit AS $= a = \frac{bf\,\mathrm{coſ}\,v}{b-f+f\,\mathrm{coſ}\,v}$.

Coroll. 3.

15. Ex natura autem ſectionum conicarum patet, ſi fuerit A orbitæ perihelium, rectarum FS & GS illam, quæ ſit brevior, propiorem eſſe perihelio A; Quare cum ſæpenumero incertum ſit, utra rectarum FS & GS deſignari debeat littera f, breviorem ſemper hac littera inſigniri conveniet.

Coroll. 4.

16. Si tangens anomaliæ v prodeat negativa, indicio hcc erit vel angulum ASF eſſe recto majorem (duobus tamen

 rectis

rectis minorem) vel eſſe negativum, atque perihelium A intra loca F & G cadere. Cum igitur poſitio perihelii A ſit anceps, & in duo loca, e diametro oppoſita incidat, ea erit vera, quæ breviori diſtantiæ f erit propior, altera vero poſitio dabit aphelium.

Problema IV.

Fig. 3. 17. Data orbita planetæ ſeu cometæ AM, invenire tempus, quo anomalia vera quævis ASM abſolvitur.

Solutio.

Sit diſtantia perihelii a ſole AS $= a$, ſemilatus rectum $= b$; & anomalia vera propoſita ASM $= v$, atque diſtantia SM $= y$, erit $y = \frac{ab}{a+(b-a)\,\mathrm{coſ}\,v}$, ſeu $\mathrm{coſ}\,v = \frac{a(b-y)}{y(b-a)}$. Sit inſuper area ASM $=$ A; erit per probl. 1. tempus quæſitum, quo planeta vel cometa ſpatium AM conficit $= \frac{A}{m\sqrt{b}}$ dierum. Quare ad tempus hoc definiendum aream ASM definiri oportet. Capiatur elementum Mm, & ducta Sm erit angulus MSm $= dv$, ideoque area trianguli MSm $= \frac{1}{2}yydv$, quæ cum ſit differentiale areæ ASM $=$ A, erit $dA = \frac{1}{2}yydv$, hincque $A = \frac{1}{2}\int yydv$, & ſi loco y ejus valor in v ſubſtituatur, erit $A = \frac{1}{2}\int \frac{aabbdv}{(a+(b-a)\,\mathrm{coſ}\,v)^2}$, quæ formula ut integrari queat, ponatur tang $\frac{1}{2}$ ASM $=$ tang $\frac{1}{2}v = t$, erit $dv = \frac{2dt}{1+tt}$; & $\mathrm{coſ}\,v = \frac{1-tt}{1+tt}$, his ſubſtitutis fit $A =$

ſaa

$\int \frac{aabbdt(1+tt)}{(b+(2a-b)tt)^2}$. Sit $A = \frac{\alpha t}{b+(2a-b)tt} + \beta \int \frac{dt}{b+(2a-b)tt}$, atque comparatione inſtituta erit $\alpha + \beta = aab$, & $\beta - \alpha = \frac{aabb}{2a-b}$ hincque $\alpha = -\frac{aab(b-a)}{2a-b}$ & $\beta = \frac{a^3 b}{2a-b}$, ex quibus colligitur area $A = \frac{a^3 b}{(2a-b)} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Hujus integralis conſtituendi ſunt quatuor caſus, quorum primus eſt:

I. Si $b = a$, quo curva fit circulus; hic caſus promtiſſime ex prima formula reſolvitur, qui dat $A = \frac{1}{2} \int aadv = \frac{1}{2} aav$; vel ſi hoc integrale per t expreſſum deſideretur ob $v = 2$ A tang. t, fiet area $A = aa$ A tang. t.

II. Sit $b > a$ ita tamen ut ſit $b < 2a$, quo caſu orbita erit ellipſis, ſitque ut vidimus $A = \frac{a^3 b}{2a-b} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Eſt vero $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{\sqrt{b(2a-b)}}$ A tang $t \sqrt{\frac{2a-b}{b}} = \frac{1}{\sqrt{b(2a-b)}}$ A ſin $\frac{t\sqrt{(2a-b)}}{\sqrt{(b+(2a-b)tt}}, = \frac{1}{\sqrt{b(2a-b)}}$ A coſ $\frac{\sqrt{b}}{\sqrt{(b+(2a-b)tt)}}$

Hinc fit $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{2\sqrt{b(2a-b)}}$ A ſin $\frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}$ ſequentes ergo prodibunt expreſſiones aream A exhibentes

A =

$$A = \frac{a^3 \sqrt{b}}{(2a-b)\sqrt{(2a-b)}} \text{ A tang } t\sqrt{\frac{2a-b}{b}} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}.$$

$$\text{vel } A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \text{ A ſin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{aa(b-a)\sqrt{b}}{2(2a-b)\sqrt{(2a-b)}}.$$

$$\frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} \text{ ſeu } A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \left(\text{A ſin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{(b-a)}{a} \cdot \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}\right)$$

III. Sit $b = 2a$, erit orbita parabola, atque erit ex priori æquatione $A = \int aadt(1+tt) = aa(t+\frac{1}{3}t^3)$, qui eſt unicus caſus, quo area algebraice poteſt exhiberi.

IV. Sit $b > 2a$ erit orbita hyperbolica, fietque $A = \frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3 b}{b-2a} \int \frac{dt}{b-(b-2a)tt}$. At hæc integratio a logarithmis pendet, fitque $\int \frac{dt}{b-(b-2a)tt} = \frac{1}{2\sqrt{b(b-2a)}}$ $l \frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}$. Ex his ergo reperitur area quæſita $A = \frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3 \sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\, l \frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}$ ſive

$$A = \frac{a^3 \sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\left(\frac{b-a}{a} \cdot \frac{2t\sqrt{b(b-2a)}}{b-(b-2a)tt} - l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}\right)$$

His igitur ſingulis caſibus ſi reperta fuerit area $ASM = A$, erit tempus, quo ea abſolvitur $= \frac{A}{m\sqrt{b}}$, quæ expreſſio tempus in diebus exhibebit, ſi fuerit $m = 271989,735$ & A & b ea menſura definiantur, quam indicavi. Q. E. J.

Coroll.

Corol. 1.

18. Si ergo orbita fuerit circulus, erit tempus, quo angulus ASM $= v$ abſolvetur $= \frac{av\sqrt{a}}{2m}$ ob $b = a$: quod quidem ex motu uniformi ſponte liquet. Ceterum quia in circulo perihelium non datur, anomaliæ veræ nomen improprie adhibetur.

Coroll. 2.

19. Si orbita fuerit ellipſis, quo area A commodius exprimatur, quæratur angulus ω, ut ſit tang. $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang. $\frac{1}{2}v = \frac{t\sqrt{(2a-b)}}{\sqrt{b}}$. Angulo autem hoc ω invento, erit area A $= \frac{a^3\sqrt{b}}{2(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\text{ſin }\omega\right)$, ideoque tempus per AM $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\text{ ſin }\omega\right)$.

Coroll. 3.

20. Si orbita fuerit hyperbola, quæratur pariter angulus ω, ut ſit tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, ſeu $t = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$. Quo angulo ω invento erit area ASM $=$ A $=$ $\frac{a^3\sqrt{b}}{2(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\text{ tang }\omega - l\text{ tang }(45° + \tfrac{1}{2}\omega)\right)$, hincque tempus per arcum AM erit $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\text{tang}\,\omega - l\text{ tang. }(45° + \tfrac{1}{2}\omega)\right)$

Coroll. 4.

21. Cum autem hic logarithmus tangentis anguli $45° + \frac{1}{2}\omega$ per integrationem ſit inventus, perſpicuum eſt, eum ex canone logarithmorum hyperbolicorum eſſe depromendum, poſito ſinu toto $= 1$. Hujusmodi vero canone deficiente, ſumatur ex tabulis vulgaribus logarithmus tang $(45° + \frac{1}{2}\omega)$, & ſubtracto 10 ab ejus characteriſta, reſiduum, multiplicetur per numerum 2, 302585092994, atque productum erit log. hyperb. tang $(45° + \frac{1}{2}\omega)$; ubi notari convenit, hujus numeri 2, 30258509 &c. logarithmum eſſe $=$ 0, 3622156886.

Scholion 1.

22. Eadem expreſſio logarithmica l tang $(45° + \frac{1}{2}\omega)$ reperitur in hydrographia, ſi quæratur in mappa Nautica ad mentem Mercatoris conſtruenda latitudo creſcens reſpondens latitudini ω in ſuperficie terræ. Quare ex tabulis nauticis, quibus latitudines creſcentes (Gall. les latitudes croiſſantes) ad ſingulos gradus exhiberi ſolent, hi valores l tang $(45° + \omega)$ ſumi poterunt. Interim tamen ſine his tabulis calculus ſatis expedite inſtituetur hoc modo. Sit id quod quæritur nempe l tang $(45° + \frac{1}{2}\omega) = x$. Tum in tabulis logarithmorum tangentium conſuetis quæratur log. tang $(45° + \frac{1}{2}\omega)$, a quo auferatur log. tang 45° ſeu 10, 00000, reſiduum ponatur $= R$, eritque $x = 2, 302585092994. R$, ideoque ſumendis logarithmis erit $lx = lR + 0, 3622156886$; unde ope tabulæ logarithmorum ſolitorum facile valor ipſius $x = l$ tang $(45° + \frac{1}{2}\omega)$ reperitur. Sit exempli gratia angulus $\omega = 37°, 22', 40''$. erit $\frac{1}{2}\omega =$

18°,

$18^{\circ}, 41', 20''$ & $45^{\circ} + \frac{1}{2}\omega = 63^{\circ}, 41', 20''$. Tum ex tabulis erit.

l tang $(45^{\circ} + \frac{1}{2}\omega)$	=	10, 3058582068
ſubtr. l tang 45°	=	10, 0000000000
erit R	=	0, 3058582068
ideoque lR	=	9, 4855201380
addatur	=	0, 3622156886
erit lx	=	9, 8477358266
unde obtinetur x	=	0, 7042645474

Scholion. 2.

23. Circa tempus in ellipſi aſſignandum notari oportet arcum ſeu angulum ω non more ſolito in gradibus & minutis, ſed in partibus radii, qui ponitur $= 1$, exprimi debere. Scilicet ſi ω eſſet 180°, tum pro ω ſubſtitui deberet longitudo ſemiperipheriæ 3, 1415926535. Hinc igitur valor cujusvis anguli ω in partibus decimalibus radii 1 exprimi poterit; convertatur arcus ω in minuta ſecunda, ſitque $\omega = n''$, atque manifeſtum eſt, hanc inſtitui debere proportionem ob $180^{\circ} = 648000''$, ut ſit $648000 : 3, 1415926535 = n :$ valorem ω, eritque ideo

$$\text{valor } \omega = \frac{3,\ 1415926535}{648000}\, n.$$

Cum

ergo ſit l 648000	=	5, 8115750059
ſubtr, l 3, 14159 &c.	=	0, 4971498727
a log. n ſubtrahatur	– – –	5, 3144251332

& reſiduum erit logarithmus ipſius ω in partibus decimalibus radii expreſſi. Vel ad logarithmum numeri n addatur conſtanter 4, 6855748668, eritque ſumma, poſtquam ejus characteriſtica fuerit denario minuta, logarithmus valoris quæſiti anguli ω.

Problema V.

Fig. 2. 24. Datis duabus a Sole diſtantiis FS & GS una cum angulo ad ſolem FSG, ac præterea tempore, quo planeta vel cometa ſpatium FG abſolvit, invenire latus rectum orbitæ, hincque ipſam orbitam determinare, ſiquidem angulus FSG fuerit minimus.

Solutio.

Sit diſtantia FS $= f$, & GS $= g$, atque angulus FSG $= \phi$ qui cum ponatur minimus, portio ſectionis conicæ FG non ſenſibiliter a linea recta diſcrepabit, erit ergo area FSG proxime triangulum rectilineum, hincque ejus area $= \frac{1}{2} fg \sin \phi$. Ob curvaturam autem arcus FG hæc area erit aliquanto major, ideoque ea propius definietur hac expreſſione $\frac{1}{2} fg \phi$, quippe quæ, ſi orbita fuerit circulus, adeo veram aream præbet. Accuratius tamen hæc area ita aſſignabitur; ſit FS $= y$, GS $= z$, & angulus ASF $= v$; tum autem ponatur area ASF $=$ F & area ASG $=$ G; erit ob ASG $= v + \phi$; area F $= \frac{1}{2} \int yy\,dv$ & G $= \frac{1}{2} \int zz\,dv$. Sit diſtantia perihelii a ſole AS $= a$, & ſemilatus rectum $= b$; erit $y = \frac{ab}{a + (b-a) \operatorname{cof} v}$ & $z = \frac{ab}{a + (b-a) \operatorname{cof} (v + \phi)}$, hinc fit $dy = \frac{ab(b-a)\,dv \operatorname{ſin} v}{(a + (b-a) \operatorname{coſ} v)^2} = \frac{(b-a) yy\,dv \operatorname{ſin} v}{ab}$, ideoque $\frac{dy}{dv} = \frac{(b-a) yy \operatorname{ſin} v}{ab}$. Cum nunc ex y naſcatur z, ſi loco v ſcribatur $v + \phi$, erit $z = y + \frac{\phi\,dy}{dv} + \frac{\phi^2\,ddy}{2\,dv^2} +$

$\phi^3 d^3 y$

$\frac{\phi^3 d^3 y}{6 dv^3}$ + &c. poſito dv conſtante. Cum autem ſimili modo ex area F naſcatur area G, ſi loco v ſcribatur $v + \phi$ erit G $= F + \frac{\phi dF}{dv} + \frac{\phi^2 ddF}{2dv^2} + \frac{\phi^3 d^3 F}{6 dv^3}$ + &c. ideoque area quæſita FSG $= G - F = \frac{\phi dF}{dv} + \frac{\phi^2 ddF}{2dv^2} + \frac{\phi^3 d^3 F}{6dv^3}$ + &c. & ob $F = \frac{1}{2} \int yy dv$ erit $\frac{dF}{dv} = \frac{1}{2} yy$; hincque porro $\frac{ddF}{dv^2} = \frac{ydy}{dv}$; $\frac{d^3 F}{dv^3} = \frac{yddy + dy^2}{dv^2}$; $\frac{d^4 F}{dv^4} = \frac{yd^3y + 3dyddy}{dv^3}$; $\frac{d^5 F}{dv^5} = \frac{yd^4y + 4dyd^3y + 3ddy^2}{dv^4}$, &c. Hinc ergo prodit $G - F = \frac{1}{2} yy\phi + \frac{\phi^2 ydy}{2dv} + \frac{\phi^3 (yddy + dy^2)}{6 dv^2} + \frac{\phi^4 (yd^3y + 3dyddy)}{24 dv^3}$ + &c. at eſt $\frac{1}{2} yz\phi = \frac{1}{2} yy\phi + \frac{\phi\phi ydy}{2dv} + \frac{\phi^3 yddy}{4dv^2} + \frac{\phi^4 yd^3y}{12dv^3}$ + &c. Quo ipſius $\frac{1}{2} yz\phi$ valore ſubtracto remanebit $G - F = \frac{1}{2}\phi yz + \frac{\phi^3 (2dy^2 - yddy)}{12dv^2} + \frac{\phi^4 (3dyddy - yd^3y)}{24dv^3} + \frac{\phi^5 (6ddy^2 + 8dyd^3y - 3yd^4y)}{240 dv^4}$ + &c. Quare cum ſit $\frac{dy}{dv} = \frac{(b-a)yy\cos v}{ab}$; erit $\frac{ddy}{dv^2} = \frac{(b-a)yy\cos v}{ab}$ +

 2(b−a)

$$\frac{2(b-a)\,y\,dy\,\mathrm{ſin}\,v}{ab\,dv} = \frac{(b-a)\,y.y\,\mathrm{coſ}\,v}{ab} + \frac{2(b-a)^2 y^3 (\mathrm{ſin}\,v)^2}{a^2 b^2};$$

$$\frac{d^3 y}{dv^3} = \frac{-(b-a)\,y\,y\,\mathrm{ſin}\,v}{ab} + \frac{6(b-a)^2 y^3\,\mathrm{ſin}\,v\,\mathrm{coſ}\,v}{a^2 b^2} + \frac{6(b-a)^3 y^4 (\mathrm{ſin}\,v)^3}{a^3 b^3};\ \&\ \frac{d^4 y}{dv^4} = -\frac{(b-a)\,y\,y\,\mathrm{coſ}\,v}{ab} - \frac{8(b-a)^2 y^3 (\mathrm{ſin}\,v)^2}{aabb} + \frac{6(b-a)^2 y^3 (\mathrm{coſ}\,v)^2}{a^2 b^2} + \frac{36(b-a)^3 y^4 (\mathrm{ſin}\,v)^2 \mathrm{coſ}\,v}{a^3 b^3} + \frac{24(b-a)^4 y^4 (\mathrm{ſin}\,v)^4}{a^4 b^4}\ \&c.$$

Ex his conficitur $\frac{2dy^2 - y\,ddy}{dv^2} = -\frac{(b-a)\,y^3\,\mathrm{coſ}\,v}{ab}$;

$$\frac{3dy\,ddy - y\,d^3y}{dv^3} = \frac{(b-a)\,y^3\,\mathrm{ſin}\,v}{ab} - \frac{3(b-a)^2 y^4\,\mathrm{ſin}\,v\,\mathrm{coſ}\,v}{a^2 b^2};$$

$$\frac{6ddy^2 + 8dy\,d^3y - 3y\,d^4y}{dv^4} = \frac{3(b-a)\,y^3\,\mathrm{coſ}\,v}{ab} + \frac{4(b-a)^2 y^4 (4(\mathrm{ſin}\,v)^2 - 3(\mathrm{coſ}\,v)^2}{a^2 b^2} - \frac{36(b-a)^3 y^5 (\mathrm{ſin}\,v)^2\,\mathrm{coſ}\,v}{a^3 b^3}.$$

Erit ergo area $G - F = \tfrac{1}{2}\Phi y z - \frac{\Phi^3 (b-a)\,y^3\,\mathrm{coſ}\,v}{12ab} + \frac{\Phi^4 (b-a)\,y^3\,\mathrm{ſin}\,v}{24ab^2} - \frac{\Phi^4 (b-a)^2 y^4\,\mathrm{ſin}\,v\,\mathrm{coſ}\,v}{8a^2 b^2} + \frac{\Phi^5 (b-a)\,y^3\,\mathrm{coſ}\,v}{80ab} + \frac{\Phi^5 (b-a)^2 y^4 (\mathrm{ſin}\,v)^2}{15a^2 b^2} - \frac{\Phi^5 (b-a)^2 y^4 (\mathrm{coſ}\,v)^2}{20a^2 b^2} - \frac{3\Phi^5 (b-a)^3 y^5 (\mathrm{ſin}\,v)^2\,\mathrm{coſ}\,v}{20a^3 b^3}$ + &c. atque $z = y + \frac{\Phi (b-a)\,y^2\,\mathrm{ſin}\,v}{ab} +$

$\Phi^2 (b-a)$

$\frac{\phi^2 (b-a)\, y\, y \,\mathrm{coſ}\, v}{2ab} + \frac{\phi^2 (b-a)^2 y^3\, (\mathrm{ſin}\, v)^2}{a^2 b^2}$. Cum igitur ſit $\mathrm{coſ}\, v = \frac{a(b-y)}{y(b-a)}$, & $\mathrm{coſ}\,(v+\phi) = \mathrm{coſ}\, v\, \mathrm{coſ}\,\phi - \mathrm{ſin}\, v.\ \mathrm{ſin}\,\phi = \frac{a(b-z)}{z(b-a)}$, erit $\mathrm{ſin}\, v = \frac{a(b-y)}{y(b-a)}.\ \mathrm{cot}\,\phi - \frac{a(b-z)}{z(b-a)}$ coſec. ϕ. Subſtituatur tantum valor pro coſ v, erit G — F $= \frac{1}{2}\phi\, y\, z - \frac{\phi^3 y y (b-y)}{12b} + \frac{\phi^4 (b-a)\, y^3\, \mathrm{ſin}\, v}{24ab} - \frac{\phi^4 (b-a)\, y^3 (b-y)\, \mathrm{ſin}\, v}{8abb}$ & loco ſin v valore ſubſtituto G — F $= \frac{1}{2}\phi\, y\, z - \frac{\phi^3 yy(b-y)}{126} + \frac{\phi^4}{24bb} y^3\ (3y - 2b) \left(\frac{b-y}{y} \mathrm{cot}\,\phi - \frac{(b-z)}{z} \mathrm{coſec.}\,\phi\right)$. Quia autem nulla eſt ratio, cur y magis in ſit quam z, hanc expreſſionem ita adornabimus, ut vero proxime ſit G — F $= \frac{1}{2}\phi\, y\, z - \frac{1}{12}\phi^3\, y\, z + \frac{\phi^3 y z \sqrt{y z}}{12b}$ rejectis terminis ſequentibus. Cum vero ſit $\mathrm{ſin}\,\phi = \phi - \frac{1}{6}\phi^3 +$ &c. erit hæc area FSG $= \frac{1}{2} y\, z\, \mathrm{ſin}\,\phi + \frac{\phi^3 y z \sqrt{y z}}{12b}$. Sit jam tempus, quo ſpatium FG abſolvitur = T dierum; erit T $= \frac{y z\, \mathrm{ſin}\,\phi}{2m\sqrt{b}} + \frac{\phi^3\, y\, z \sqrt{y z}}{12mb\sqrt{b}}$ exiſtente $m = 271989, 735$. Quoniam ſin ϕ non multum a ϕ diſcrepat, ponatur T $= \frac{y z\, \mathrm{ſin}\,\phi}{2m\sqrt{b}} + \frac{y z \sqrt{y z}}{12mb\sqrt{b}} (\mathrm{ſin}\,\phi)^3$,

& for-

& formetur ad b inveniendum hæc æquatio $\frac{\sqrt{yz}}{\sqrt{b}}$ ſin $\Phi =$ $\frac{2mT}{\sqrt{yz}} + Q$, fietque $T = T + \frac{Q\sqrt{yz}}{2m} + \frac{2m^2T^3}{3yz\sqrt{yz}}$, hincque $Q = \frac{-4m^3T^3}{3y^2z^2}$. Ex his obtinetur $\frac{\sqrt{yz}}{\sqrt{b}}$ ſin $\Phi =$ $\frac{2mT}{\sqrt{yz}} - \frac{4m^3T^3}{3y^2z^2}$, & $\frac{\text{ſin}\,\Phi}{\sqrt{b}} = \frac{2mT}{yz} - \frac{4m^3T^3}{3y^2z^2\sqrt{yz}}$. Erit ergo $\frac{\sqrt{b}}{\text{ſin}\,\Phi} = \frac{yz}{2mT} + \frac{mT}{3\sqrt{yz}}$, ideoque ſemilatus rectum $b = \left(\frac{yyzz}{4m^2T^2} + \frac{1}{3}\sqrt{yz}\right)(\text{ſin}\,\Phi)^2$. Quo cognito orbita per probl. III definietur. Q. E. J.

Coroll. 1.

25. Si ergo cometæ duæ fuerint cognitæ a Sole diſtantiæ $FS = f$, & $GS = g$, una cum angulo $FSG = \Phi$ ac præterea obſervatum ſit tempus per ſpatium FG, quod ſit $= T$ dier. erit orbitæ ſemilatus rectum $b = \left(\frac{ffgg}{4m^2T^2} + \frac{1}{3}\sqrt{fg}\right)$ $(\text{ſin}\,\Phi)^2$, calculus autem ſæpe facilius inſtituetur quærendo $\sqrt{b} =$ $\left(\frac{fg}{2mT} + \frac{mT}{3\sqrt{fg}}\right)$ ſin Φ.

Coroll. 2.

26. Hinc ex cognito tempore T area FSG propius definiri, & quantum ſuperet triangulum rectilineum FSG ducta corda FG comprehenſum, aſſignari poteſt. Exceſſus ſcilicet

ſcilicet erit ſegmentum FG inter arcum curvæ & cordam contentum, quodeſt $= \frac{fg\sqrt{fg}}{12b}(\text{ſin}\,\phi)^3 = \frac{m^2T^2\,\text{ſin}\,\phi}{3\sqrt{fg}}$

Coroll. 3.

27. Cognito hoc modo latere recto, cujus ſemiſſis $= b$, ſtatim reperietur poſitio perihelii A; poſito enim angulo ASF $= v$, erit tang $v =$ cot $\phi - \frac{(b-g)f}{(b-f)\,\text{ſin}\,\phi}$; hincque porro definitur diſtantia perihelii a ſole AS $= a = \frac{bf\,\text{coſ}\,v}{b-f+f\,\text{coſ}\,v}$.

Coroll. 4.

28. Inventis ergo anomalia vera ASF, & rectis a & b aſſignari poterit tempus, quod ad ſpatium AF abſolvendum impenditur, hincque cum datum ſit tempus, quo planeta vel cometa in loco F hæſit, definiri poterit temporis momentum, quo in perihelio eſt verſatus.

Scholion 3.

29. Ad hoc ergo tempus definiendum inſerviet methodus in probl. IV. expoſita, cujus tres conſtituendi ſunt caſus, quorum primus locum habet, ſi curva AFG fuerit ellipſis ſeu $2a > b$. Hoc caſu quæratur angulus ω, ita ut ſit tang $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang $\frac{1}{2}v$; hocque cognito erit tempus per ſpatium AF $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\,\text{ſin}\,\omega\right)$ dierum. Alter caſus adhiberi debet, ſi curva AFG fuerit hyperbola, tum ob

$b > 2a$ quæratur angulus ω, ut ſit $\text{tang}\,\frac{1}{2}\,\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$. $\text{tang}\,\frac{1}{2}\,v$. quo cognito erit tempus per $AF = \frac{a^3}{2m\,(b-2a)^{\frac{3}{2}}}\left(\frac{b-a}{a}\,\text{tang}\,\omega - l\,\text{tang}\,(45^\circ + \frac{1}{2}\,\omega)\right)$. Tertius caſus pro curva AFG parabola eſt conſiderandus, ſi ſit $b = 2a$, tum ſumto $t = \text{tang}\,\frac{1}{2}\,v$, erit tempus per $AF = \frac{a\sqrt{a}}{m\sqrt{2}}\,(t + \frac{1}{3}\,t^3)$ dierum. Quemadmodum autem calculus pro parabola eſt facillimus, ita idem maxime impeditur, ſi orbita tantum proxime ad parabolam acceſſerit, Dum enim $2a - b$ vel $b - 2a$ quantitas minima evadit, cum angulus ω ſit nimis parvus, tum vero in temporis expreſſione denominator tam exiguus, ut minimus error in angulo ω commiſſus maximam aberrationem in tempore parere poſſit. Hanc ob rem his caſibus, quibus orbita cometæ proxime ad parabolam accedit, conveniet tempus idonea approximatione exhibitum potius quam verum uſurpare.

Problema VI.

Fig. 3. 30. Si orbita Cometæ non multum a parabola diſcrepet ſive ſit ellipſis ſive hyperbola, definire tempus, quo data quævis. Anomalia vera ASM conficitur.

Solutio.

Sit ut hactenus diſtantia $AS = a$, & ſemi-latus rectum $= b$, quoniam $2a$ & b non multum a ſe invicem diſcrepant, pona-

poſnatur $2a - b = \delta$, erit δ quantitas minima, & affirmativa quidem, ſi curva fuerit ellipſis, at negativa ſi curva ſit hyperbola. Jam ſit anomalia vera propoſita ASM $= v$, ac ponatur tang $\frac{1}{2} v = t$, erit ut ſupra (17) vidimus area ASM $= \frac{\int aabbdt(1+tt)}{(b+\delta tt)^2}$; quæ ſtatuatur $= \frac{aabbz}{b+\delta tt}$, atque ob $b + \delta = 2a$ reperietur hic valor pro $z = \frac{t}{b} + \frac{2at^3}{3bb} - \frac{2a\delta t^5}{3.5b^3} + \frac{2a\delta^2 t^7}{5.7b^4} - \frac{2a\delta^3 t^9}{7.9b^5}$ &c. ergo area ASM $=$

$$\frac{aab}{b+\delta tt}\left(t + \frac{2at^3}{3b} - \frac{2a\delta t^5}{3.5b^2} + \frac{2a\delta^2 t^7}{5.7b^2} - \frac{2a\delta^3 t^9}{7.9b^4} + \&c.\right)$$

Vel cum ſit $\frac{b}{b+\delta tt} = 1 - \frac{\delta tt}{b} + \frac{\delta^2 t^4}{bb} - \frac{\delta^3 t^6}{b^3} + \&c.$

erit area ASM $= aa\Big(t + \frac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta\delta}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9$ &c.
$\qquad + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\Big]$

Quare cum tempus ſit $= \frac{\text{ar. ASM}}{m\sqrt{b}}$ dierum, erit tempus quo anomalia vera ASM $= v$ abſolvitur in diebus expreſſum $= \frac{aa}{m\sqrt{b}}\Big(t + \frac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta^2}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9$ &c.
$\qquad + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\Big]$

quæ expreſſio, etſi in infinitum progreditur, tamen citiſſime conver-

convergit, ſi $\delta = 2a - b$ fuerit quantitas vehementer parva, uti ſtatuimus. Q. E. J.

Coroll. 1.

31. Quia eſt $2a = b + \delta$, ſi in ſerie infinita hic valor ubique loco $2a$ ſubſtituatur, reperietur tempus, quo anomalia vera ASM $= v$, cujus ſemiſſis tangens ponitur $= t$, abſolvitur

$$= \frac{aa}{m\sqrt{b}} \left(t + \tfrac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \right. \;\&c.$$
$$\left. + \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \right]$$

Coroll. 2.

32. Quoniam pro hyperbola fit δ numerus negativus, omnes termini prodibunt affirmativi, ſi enim pro hyperbola ponatur $b - 2a = \delta$ erit tempus per arcum AM $= \frac{aa}{m\sqrt{b}}$

$$\left(t + \tfrac{1}{3}t^3 + \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 + \frac{4\delta^3}{9b^3}t^9 \right. \;\&c.$$
$$\left. + \frac{3\delta\delta}{5bb}t^5 + \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \right]$$

Coroll. 3.

33. Series hæ maxime convergunt, quo minor fuerit angulus ASM; ſin autem hic angulus v fiat tantus, ut ejus ſemiſſis multum ſuperet ſemirectum, ideoque ejus tangens t unitatem longe ſuperet, tum convergentia diminuetur. His ergo caſibus expediet methodo directa uti. Quando quidem angulus ille ω ita ſumtus, ut ſit tang $\frac{1}{2}\,\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$

tang

tang $\frac{1}{2} v$, vel tang $\frac{1}{2} \omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2} v$, prodit adhuc notabilis magnitudinis.

Problema VII.

34. Si orbita cometæ non admodum a parabola diſcrepet, ex dato tempore, quod vel ante vel poſt appulſum ad perihelium elapſum ſit, locum cometæ in Orbita, hoc eſt anomaliam veram ASM, una cum diſtantia cometæ a ſole invenire. Fig. 3

Solutio.

Quoniam primo orbita cometæ eſt data, ponatur diſtantia perihelii a ſole A S $= a$, ſemilatus rectum $= b$; & quoniam orbita non multum a parabola differre ſtatuitur poſito $2a - b = \delta$, erit δ quantitas valde parva. Deinde ſit tempus, quod vel ante appulſum ad perihelium effluxerit, vel poſt, in diebus expreſſum $=$ T; Anomalia vera autem, quæ quæritur, ASM ſit $= v$, ac $t =$ tang $\frac{1}{2} v$. Jam ex præcedente problemate habebimus hanc æquationem

$$T = \frac{aa}{m\sqrt{b}}\left(t + \tfrac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \quad \&c.\right. \\ \left. + \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \right]$$

ex qua valorem ipſius t erui oportebit. Ponamus $\frac{mT\sqrt{b}}{aa} = n$ brevitatis gratia, ut ſit $n = t + \frac{1}{3}t^3$

 -2δ

$$-\frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9$$
$$+\frac{3\delta\delta}{5bb} \quad -\frac{4\delta^3}{7b^3} \quad +\frac{5\delta^4}{9b^4}$$ &c. Sit primum orbita cometæ vera parabola, ideoque $\delta = o$, erit $n = t + \frac{1}{3}t^3$, atque $t^3 + 3t - 3n = o$; cujus æquationis cubicæ per regulam Cardani radix erit:

$$t = \sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})} - \sqrt[3]{(-\tfrac{3}{2} + \sqrt{(\tfrac{9}{4}nn + 1)})}$$ vel

$$t = \sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})} - \frac{1}{\sqrt[3]{(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}}$$ vel etiam

$$t = \frac{1}{\sqrt[3]{(-\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}} - \sqrt[3]{(-\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)})}$$

Vel igitur ex his formulis, vel ex tabulis in hunc finem computatis valor ipſius t erui poterit, quo cognito dabitur anomalia vera ASM $= v$, per æquationem tang $\frac{1}{2}v = t$. Deinde autem habebitur diſtantia cometæ a ſole SM $= y = \frac{ab}{a + (b-a)\operatorname{coſ} v}$. Cum vero pro parabola ſit $b = 2a$; erit $y = \frac{2a}{1 + \operatorname{coſ} v} = \frac{a}{(\operatorname{coſ}\frac{1}{2}v)^2}$. Sic itaque ſi orbita cometæ vera fuerit parabola, determinabitur ad datum tempus locus cometæ in orbita.

Sin autem orbita cometæ fuerit ellipſis maxime excentrica vel hyperbola non multum a parabola abhorrens, ita ut ſit $\delta = 2a - b$ quantitas vehementer parva, poſito $\frac{mT\sqrt{b}}{aa} = n$, hæc æquatio reſolvi debebit:

$n = t$

$$n = t + \tfrac{1}{3}t^3 - \frac{2\delta}{5b} t^5 + \frac{3\delta\delta}{7bb} t^7 - \frac{4\delta^3}{9b^3} t^9 \ \&c.$$
$$\quad + \frac{3\delta\delta}{5bb} - \frac{4\delta^3}{7b^3} + \frac{5\delta^4}{9b^4}$$

Sumatur primo tantum hæc æquatio $n = t + \frac{1}{3} t^3$, & per modum præcedentem quæratur valor ipſius t, ſitque $t = \theta$, ita ut ſit $n = \theta + \frac{1}{3}\theta^3$; eritque ob δ valde parvam quantitatem θ valor ipſius t vero proximus. Sit igitur verus valor $t = \theta + A\theta^3 + B\theta^5 + C\theta^7 + \&c.$ erit

$$t^3 = \theta^3 + 3A\theta^5 + 3B\theta^7 \ \&c.$$
$$\qquad\qquad\qquad + 3A^2\theta^7$$

$$t^5 = \theta^5 + 6A\theta^7, \&c. \quad \& \ t^7 = \theta^7.$$

quibus ſubſtitutis in æquatione orietur $n = \theta + \frac{1}{3}\theta^3 =$

$$\begin{array}{llll} \theta + A\theta^3 & + B\theta^5 & + C\theta^7 & \\ \quad + \frac{1}{3}\theta^3 & + A\theta^5 & + B\theta^7 & + \&c. \\ & - \frac{2\delta}{5b}\theta^5 & + A^2\theta^7 & \\ & + \frac{3\delta\delta}{5bb}\theta^5 & - \frac{2\delta}{b}A\theta^7 & \&c. \\ & & + \frac{3\delta\delta}{bb}A\theta^7 & \\ & & + \frac{3\delta^3}{7bb}\theta^7 & \\ & & - \frac{4\delta^3}{7b^3}\theta^7 & \end{array}$$

Singulis ergo terminis more ſolito ad nihilum reductis erit

A = o;

$A = o$; $B = \frac{2\delta}{5b} - \frac{3\delta\delta}{5bb}$; $C = -B - \frac{3\delta\delta}{7bb} + \frac{4\delta^3}{7b^3} =$
$-\frac{2\delta}{5b} + \frac{6\delta\delta}{35bb} + \frac{4\delta^3}{7b^3} = \frac{2\delta}{b}\left(\frac{\delta}{b} + 1\right)\left(\frac{2\delta}{7b} - \frac{1}{5}\right)$

Cognito ergo ipſius t valore vero proximo θ, erit verus valor

$$t = \theta + \left(\frac{2\delta}{5b} - \frac{3\delta\delta}{5bb}\right)\vartheta^5 - \left(\frac{2\delta}{5b} - \frac{6\delta\delta}{35bb} - \frac{4\delta^3}{7b^3}\right)\vartheta$$

Tum vero ob $t =$ tang $\frac{1}{2}$ v cognoscetur anomalia vera ASM $= v$; hincque porro prodibit diſtantia cometæ a ſole

$$SM = y = \frac{ab}{a + (b-a)\cos v} = \frac{b}{1 + \frac{b-a}{a}\cos v}. \quad \text{Q. E. J.}$$

Coroll. 1.

35. Pro ellipſi ergo maxime oblonga δ ob quantitatem affirmativam, erit verus valor ipſius t major quam valor θ, qui ex hypotheſi orbitæ parabolicæ eſt erutus.

Coroll. 2.

36. Pro hyperbola autem ubi ſit δ quantitas negativa verus valor ipſius t minor erit quam θ;. utroque autem caſu ob tantas ipſius θ poteſtates, expreſſio pro t vehementer convergit, ſiquidem ſit $\theta < 1$, quod ſit ſi anomalia vera v fuerit angulo recto minor.

Scholium.

37. Quod ſi autem anomalia vera v multum excedat angulum rectum, ita ut δ vel t fiat numerus unitate multo major, hincque expreſſio illa pro t inventa divergat potius quam convergat, tum iſta methodo uti non conveniet, niſi forte δ ſit

quan-

quantitas tantopere exigua, ut per eam omnes termini efficiantur minimi. Oportebit igitur his caſibus expreſſionem veram, qua relatio inter tempus & anomaliam veram exhibetur, in ſubſidium vocare; atque ex ea modum, quo ad datum tempus anomalia vera aſſignari poſſit, eruere. Hanc itaque operationem in ſequentibus problematis fuſius ſum oſtenſurus, ut quovis caſu oblato theoria adhiberi, per eamque verus motus tam cometarum quam planetarum definiri queat.

Problema VIII.

38. Si orbita planetæ vel cometæ fuerit ellipſis quæcunque cognita, ad datum tempus, vel ante vel poſt appulſum ejus ad perihelium A, ejus anomaliam veram ASM, atque diſtantiam a ſole SM aſſignare.

Solutio.

Sit perihelii a ſole diſtantia AS $= a$, & ſemilatus rectum $= b$; erit $2a > b$, quia orbita ponitur ellipſis; Tempus autem, quod vel ante vel poſt appulſum ad perihelium effluxerit, ſit in diebus expreſſum $=$ T. Nunc ponatur anomalia vera, quæ quæritur ASM $= v$, ſitque alius angulus ω ita comparatus ut ſit tang $\frac{1}{2}\,\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang $\frac{1}{2}\,v$, eritque ut vidimus (19), tempus $T = \frac{a^3}{2m\,(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\,\text{ſin}\,\omega\right)$ ex qua æquatione angulum ω erui oportet. Fiet ergo $\omega - \frac{(b-a)}{a}$

 ſin $\omega =$

$\sin \omega = \frac{2\, m\, (2\, a - b) \frac{3}{2}\, T}{a^3}$. Convertatur hæc expreſſio $\frac{2m\,(2a-b)\frac{3}{2}\,T}{a^3}$ quæ eſt cognita, in arcum circuli cujus radius $= 1$, quod ita fiet: Sumatur expreſſionis $\frac{2m\,(2a-b)\frac{3}{2}\,T}{a^3}$ logarithmus, ad quem addatur conſtanter hic logarithmum 5, 3144251332; atque ſumma erit logarithmus anguli hujus quæſiti in minutis ſecundis expreſſi. Vel cum ſit logarithmus $m = 5,4345525139$, ad log. $\frac{(2a-b)\frac{3}{2}\,T}{a^3}$ addatur conſtanter iſte logarithmus 11, 0500076428, & numerus logarithmo reſultanti reſpondens dabit angulum in minutis ſecundis expreſſum. Sit iſte angulus $= u$, qui erit ille ipſe, quem Aſtronomi anomaliam mediam vocare ſolent, qui ergo hoc pacto ex dato tempore T inveniri, vel ex tabulis aſtronomicis, ſi quæſtio de planeta quopiam inſtituatur, depromi poteſt. Priori autem modo ſi iſte angulus u in minutis ſecundis exprimatur erit $l\,u = 11,0500076428 + \frac{3}{2}\,l(2a-b) + l\,T - 3\,l\,a$, unde deficientibus tabulis aſtronomicis, anomalia media u expedite reperitur. Inventa erga jam ſit hæc anomalia media u, eritque $u = \omega - \frac{(b-a)}{a} \sin \omega$. Hæc æquatio commodiſſime aliquot tentaminibus reſolvetur. Scilicet cum ſit $\omega > u$, ſumatur pro arbitrio angulus pro ω, & computetur angulus $= \frac{(b-a)}{a}$

ſin ω, quod fiet, ſi a $l\,\frac{b-a}{a} + l$ ſin ω ex tabulis ſumto ſubtrahatur 4, 6855748668, logarithmus enim reſiduus dabit numerum minutorum ſecundorum ipſi $\frac{(b-a)}{a}$ ſin ω æquivalentium; hoc igitur facto pro $\omega - \frac{(b-a)}{a}$ ſin ω reperietur angulus vel major vel minor quam u; priori caſu valor pro ω aſſumtus erat nimis magnus, poſteriori vero nimis parvus; ſic igitur corrigendo hypotheſin factam mox angulus ω prope verus cognoſcetur. Sit valor jam prope verus pro ω inventus $= \varrho$; verus autem valor ſit $\omega = \varrho + z$, erit ſin $(\varrho + z) =$ ſin ϱ $+ z$ coſ ϱ; ideoque $u = \varrho + z - \frac{(b-a)}{a}$ ſin $\varrho - (b-u)\, z$ coſ ϱ & hinc naſcitur $z = \frac{u - \varrho + \frac{(b-a)}{a} \sin\varrho}{1 - \frac{(b-a)}{a} \cos\varrho}$, ubi in numeratore pars $\frac{b-a}{a}$ ſin ϱ pari modo in angulum converti debet, ut ſupra oſtendimus. Denominator autem erit merus numerus, & ob ſinum totum $= 1$, logarithmi coſ ϱ characteriſtica denario minui debet. Hoc ergo modo reperitur angulus z, qui ad angulum ϱ additus dabit verum angulum quæſitum ω. Sin autem angulus ϱ aſſumtus nimium a vero diſceſſerit, hoc modo multo propior valor pro ω orietur, qui tum loco ϱ ſubſtitutus vero proximum valorem pro ω ſuppeditabit. Quodſi au-

tem hac methodo repertus fuerit angulus ω, hinc facile invenietur anomalia vera ASM $= v$ ope hujus formulæ tang $\frac{1}{2} v$ $= \frac{\sqrt{b}}{\sqrt{(2a-b)}}$ tang $\frac{1}{2}\omega$. Tandem vero erit diſtantia planetæ ſeu cometæ a ſole SM $= \frac{ab}{a+(b-a)\operatorname{coſ} v}$. Q. E. J.

Coroll. 1.

39. Coefficiens $\frac{b-a}{a}$, qui in hoc calculo occurrit, vocari ſolet orbitæ excentricitas; (9) hujus ergo logarithmus imprimis notari debet, quippe quo cognito totus calculus facillime inſtituetur.

Coroll. 2.

40. Ex data ergo orbitæ excentricitate $\frac{b-a}{a}$ hujus calculi ope ad anomaliam mediam quamvis propoſitam expedite anomalia vera reſpondens definietur: Hocque modo tabulæ æquationis motuum planetarum & cometarum, qui quidem in ellipſibus revolvuntur, ſupputabuntur.

Problema IX.

Fig. 3. 41. Si cometa in hyperbola circa ſolem moveatur, atque tempus, quo per perihelium A tranſit, fuerit notum, ad datum quodvis tempus locum cometæ in orbita ſeu anomaliam veram ASM, ejusque a ſole diſtantiam SM ſupputare.

Solutio.

Solutio.

Sit iterum perihelii a ſole diſtantia AS $= a$, & ſemilatus rectum $= b$, eritque $b > 2a$. Tempus autem propoſitum a momento, quo Cometa in perihelio hæret, diſtet intervallo $=$ T dierum. Deinde ſit anomalia vera, quam inveniri oportet, ASM $= v$; ac concipiatur alius angulus ω, ut ſit tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$; eritque (20), T $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{b-a}{a}\text{ tang }\omega - l\text{ tang }(45^\circ + \frac{1}{2}\omega)\right)$. Statuatur $\frac{2m(b-2a)^{\frac{3}{2}}T}{a^3} = u$, erit u quantitas cognita, atque illi, quæ ante anomaliæ mediæ nomine occurrerat, analoga: hoc vero caſu expediet u in meris numeris expreſſam retinere, quam in angulum convertere. His monitis erit itaque $u = \frac{b-a}{a}$ tang $\omega - l$ tang $(45^\circ + \frac{1}{2}\omega)$ unde tentando pro ω ejusmodi valor eruatur qui a vero non multum diſſideat. Sit ergo ϱ angulus non multum a vero anguli ω valore diſcrepans, ac ponatur $\omega = \varrho + z$, ob z angulum valde parvum erit tang. $\omega = \frac{\text{tang }\varrho + z}{1 - z\text{ tang }\varrho}$ ſeu tang $\omega =$ tang $\varrho + z(1 + (\text{tang }\varrho)^2)$ $=$ tang $\varrho + \frac{z}{(\text{coſ }\varrho)^2}$. Ad l tang $(45^\circ + \frac{1}{2}\omega)$ inveniendum ponatur l tang $(45^\circ + \frac{1}{2}\varrho) =$ R, eritque l tang $(45^\circ + \frac{1}{2}\omega)$ valor ipſius R, qui prodit ſi loco ϱ ſcribatur $\varrho + z$, prodit

prodit autem is $= R + \frac{z\, dR}{d\varrho}$. Erit vero $dR =$

$$\frac{\frac{1}{2} d\varrho}{(\text{coſ}(45^\circ + \frac{1}{2}\varrho))^2 \text{tang}(45^\circ + \frac{1}{2}\varrho)} = \frac{d\varrho}{\text{coſ}\,\varrho};$$ ideoque $l\,\text{tang}$

$(45^\circ + \frac{1}{2}\omega) = l\,\text{tang}(45^\circ + \frac{1}{2}\varrho) + \frac{z}{\text{coſ}\,\varrho}$. His ſubſtitutis erit

$$u = \frac{b-a}{a}\,\text{tang}\,\varrho + \frac{(b-a)z}{a(\text{coſ}\varrho)^2} - l\,\text{tang}\,(45 + \tfrac{1}{2}\varrho) - \frac{z}{\text{coſ}\,\varrho},$$

unde naſcitur $z = \dfrac{u + l\,\text{tang}\,(45^\circ + \frac{1}{2}\varrho) - \frac{(b-a)}{a}\,\text{tang}\,\varrho}{\frac{1}{(\text{coſ}\varrho)^2}\left(\frac{b-a}{a} - \text{coſ}\,\rho\right)}$

ſeu $z = \dfrac{u + l\,\text{tang}\,(45 + \frac{1}{2}\varrho) - \frac{(b-a)}{a}\,\text{tang}\,\varrho}{\frac{(b-a)}{a} - \text{coſ}\,\rho}(\text{coſ}\varrho)^2$. Hoc modo reperietur numerus pro z, dummodo ea regula ad $l\,\text{tang}$ $(45 + \frac{1}{2}\rho)$ exprimendum obſervetur, quæ ſupra (22) eſt tradita. Hic autem numerus pro z inventus in angulum eſt convertendus, modo, quem ante expoſui. Hisque factis habebitur verus valor anguli $\omega = \varrho + z$, qui autem, ſi adhuc dubium ſuperſit, denuo pro ρ ipſe poni, ſicque ulterius corrigi poterit. Invento autem angulo ω, ſtatim reperietur $\text{tang}\,\frac{1}{2}v = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ $\text{tang}\,\frac{1}{2}\omega$, atque adeo definietur anomalia vera quæſita ASM $= v$, ex qua porro elicitur diſtantia cometæ a ſole SM $= y = \frac{ab}{a + (b-a)\,\text{coſ}\,v}$. Q. E. J.

Scholion.

Scholion.

42. Ad uſum hujus calculi, qui minus eſt uſitatus in aſtonomia, uberius declarandum, ponamus cometam in hyperbola æquilatera deferri, atque perihelii ejus à ſole diſtantiam eſſe = diſtantiæ terræ a ſole mediæ, nempe eſſe . . . . $a = 100000$
unde fiet $b = a(1 + \sqrt{2})$, ſeu $b = 241421,356$
& quæri debeat locus hujus cometæ centum diebus poſt tranſitum ejus per perihelium; ita ut ſit $T = 100$. Primo ergo quæri debet u ut ſit $l\,u = l\,2\,m + l\,T + \frac{1}{2}\,l\,41421,356 - 3\,l\,100000$; unde ſequens calculus naſcitur.

$l\,2\,m$	=	5,7355825
$l\,T$	=	2,0000000
$l\,41421,356$	=	4,6172243
ſemiſſ.	=	2,3086122
		14,6614190
$3\,l\,100000$	=	15,0000000
$l\,u$	=	9,6614190
unde u	=	0,458584

Deinde eſt $\frac{b-a}{a} = 1,41421356 = \sqrt{2}$

ideoque $l\,\frac{b-a}{a} = 0,1505150$

Statuatur primum $\tang \omega = \frac{a}{b-a}\,u$; neglecto altero termino

erit ob $l\,u$	=	9,6614190
ſubtr. $l\,\frac{b-a}{a}$	=	0,1505150
$l\,\tang \omega$	=	9,5109040

ideoque

ideoque $\omega = 17°, 58'$,
& hinc fiet angulus $45° + \frac{1}{2}\omega = 53°, 59'$,
cujus tangens eſt $= 1,3755403$.

Patet ergo angulum $17°,58'$ vehementer nimis eſſe parvum & ſi pro ω ſumatur $30°$, adhuc iſte valor nimis erit parvus, prodit enim pro u hic numerus $0,26719$, ſin autem ſtatuatur $\omega = 40°$, prodit pro u iſte numerus $0,4237$, quare cum ſit $u = 0,458584$, perſpicuum eſt hunc valorem non multum a veritate abludere, atque ex his duabus poſitionibus $30°$ & $40°$ conjecturam faciendo reperietur ω prope modum eſſe debere $42°$, qui valor ergo pro ϱ accipiatur, ita ut ſit $\varrho = 42°$, & $\frac{1}{2}\varrho = 21°$ atque $45° + \frac{1}{2}\varrho = 66°$. Calculus itaque ſic inſtituatur

l tang $66° - 10$	$=$	$0,3514169$
hujus log.	$=$	$9,5458226$
addatur per (22)	...	$0,3622157$
Erit ergo		$9,9080383$
l tang $(45° + \frac{1}{2}\varrho)$	$=$	$0,809167$
Porro eſt l tang ϱ	$=$	$9,9544374$
add. $l\,\frac{b-a}{a}$	$=$	$0,1505150$
$l\,\frac{b-a}{a}$ tang ϱ	$=$	$1,1049524$
Ergo $\frac{b-a}{a}$ tang ϱ	$=$	$1,273364$

Quo circa ad u	$=$	$0,458584$
addatur l tang $(45° + \frac{1}{2}\varrho)$	$=$	$0,809167$
		$1,267751$
ſubtrahatur $\frac{b-a}{a}$ tang ϱ	$=$	$1,273364$
erit numerator	$=$	$-0,005613$

$a = b$

$\frac{b-a}{a}$	=	1, 4 1 4 2 1 3 5
$\cos\varrho$	=	7 4 3 1 4 9 8
Denom.	=	0, 6 7 1 0 6 8 7
Jam eſt $l\cos\varrho$	=	9, 8 7 1 0 7 3 5
dupl. $l(\cos\varrho)^2$		9, 7 4 2 1 4 7 0
Add. l num.		7, 7 4 9 1 9 5 0
		7, 4 9 1 3 4 2 0
ſubtr. l den.	=	9, 8 2 6 7 6 7 0
$l-z$	=	7, 6 6 4 5 7 5 0
ſubtr.		4, 6 8 5 5 7 4 9
		2, 9 7 9 0 0 0 1

ergo $-z = 952, 79'' = 15', 53''$

Erit ergo $\omega = \rho + z = 41°, 44', 7''$.

Invento ergo angulo $\omega = 41°, 44', 7''$, erit tang $\frac{1}{2}v = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$: at ob $b = a(1+\sqrt{2})$ erit $\frac{b}{b-2a} = \frac{1+\sqrt{2}}{\sqrt{2}-1} = (1+\sqrt{2})^2$ ideoque $\sqrt{\frac{b}{b-2a}} = 1+\sqrt{2} = 2,41421356$, & ob $\frac{1}{2}\omega = 20°, 52', 3\frac{1}{2}''$ calculus ita ſe habebit.

Ad. $l\sqrt{\frac{b}{b-a}}$	=	0, 3 8 2 7 7 5 6
add. l tang $\frac{1}{2}\omega$	=	9, 5 8 1 1 7 0 9
l tang $\frac{1}{2}$ v	=	9, 9 6 3 9 4 6 5
Ergo $\frac{1}{2}$ v	=	42°, 37', 28''
Ac propterea v	=	85°, 14', 56''

Atque ideo cometa iſte tempore 100 dierum poſt tranſitum per perihelium deſcripſit angulum ASM $= v = 85°$,

14′, 56″. Erit ergo hoc tempore ejus a ſole diſtantia SM =

$$y = \frac{ab}{a + (b - a) \operatorname{coſ} v} = \frac{b}{1 + \frac{b-a}{a} \operatorname{coſ} v}.$$

Quare ad $l \operatorname{coſ} v$	=	8, 9 1 8 1 7 4 7
add. $l \frac{b-a}{a}$	=	0, 1 5 0 5 1 5 0
		9, 0 6 8 6 8 9 7
ergo $\frac{b-a}{a} \operatorname{coſ} v$	=	0, 1 1 7 1 3 6
& denom.	=	1, 1 1 7 1 3 6
Jam a $l\,b$	=	5, 3 8 2 7 7 5 6
ſubtr. l denom.	=	0, 0 4 8 1 0 6 1
erit $l\,y$	=	5, 3 3 4 6 6 9 5
& SM = y	=	2 1 6 1 0 7.

Cum igitur diſtantia a Sole in perihelio eſſet 100000, erit poſt 100 dies cometæ a ſolo diſtantia = 216107. Hocque exemplum ad calculum hujus methodi illuſtrandum ſufficiet.

Problema X.

Fig. 4. 43. Datis duobus planetæ cometæve locis F & H a ſe invicem non multum remotis una cum tempore, quo ſpatium FH eſt confectum, ad quodyis tempus medium ejus locum verum G aſſignare.

Solutio.

Quo hoc problema commodius reſolvere queamus, applicemus id ad motum terræ, quam motu medio in diſtantia a ſole mediocri circulum percurrere concipiamus. Exiſtente ergo ſole in s ſint f & h duo terræ loca, & tempus per fh ſit = T dierum.

dierum. Sit diſtantia media terræ a ſole $fs = bs = c = 100000$ ut hactenus ſumſimus, hincque reperietur angulus fsb, ad $l\,\frac{2m\,T}{c\sqrt{c}}$ addendo 5,3144251332, numerus enim reſpondens ſummæ exhibebit angulum fsb in minutis ſecundis, ſeu ob m & c in numeris data, ad l T addatur hic logarithmus 2, 5500076427, & numerus ſummæ reſpondens dabit angulum fsb in minutis ſecundis expreſſum. Diſpertiatur jam tempus T in duas partes α, & β ita ut ſit $T = \alpha + \beta$ atque elapſa temporis parte α manifeſtum eſt terram futuram eſſe in g, ita ut ducta gs, ſit ang. $fsg : gsb = \alpha : \beta$; erit ergo $fsg = \frac{\alpha}{T}\, fsb$, & $gsb = \frac{\beta}{T}\, fsb$. Ducatur corda fb, ſecans radium sg in o, & quæratur ſagitta og hoc modo: ob ang. $sfo = 90^\circ - \frac{1}{2} fsb$, & $sof = 90 + \frac{1}{2} fsb - \frac{\alpha}{T} fsb = 90 + \frac{(\beta - \alpha)}{2T} fsb$, erit $so : sf = \text{cof}\,\frac{1}{2} fsb : \text{cof}\,\frac{\beta - \alpha}{2T}\, fsb$, hincque $so = c\,\frac{\text{cof}\,\frac{1}{2} fsb}{\text{cof}\,\frac{\beta - \alpha}{2T} fsb}$; ac propterea ſagitta $go =$

$$\frac{c\left(\text{cof}\,\frac{\beta - \alpha}{2T}\, fsb - \text{cof}\,\frac{1}{2}\, fsb\right)}{\text{cof}\,\frac{\beta - \alpha}{2T}\, fsb} = \frac{2c\,\text{ſin}\,\frac{\beta}{2T}\, fsb.\ \text{ſin}\,\frac{\alpha}{2T}\, fsb}{\text{cof}\,\frac{\beta - \alpha}{2T}\, fsb}$$

Eſt vero $\frac{1}{2T}\, fsb$ angulus conſtans, qui erit $= 29', 34$

 98″

$\frac{98''}{1000}$ ſeu $= 1774, 098''$, cujus numeri logarithmus eſt 3, 2489776471. Quodſi ergo hic angulus $\frac{1}{2T} f s h = 29', 34 \frac{''98}{1000}$ ponatur $= \tau$, & tempora α & β in diebus exprimantur, erit ſagitta $g o = \frac{2c \sin \beta\tau . \sin \alpha\tau}{\cos (\beta - \alpha)\tau}$. His præmiſſis obſervatus ſit planeta ſeu cometa in F tempore autem $T = \alpha + \beta$ elapſo in H, & quæratur ubi is ſit futurus elapſo tempore tantum α, poſtquam in F eſt verſatus. Sit G locus quæſitus, & quoniam tempora in eadem orbita ſunt ut areæ circa ſolem S deſcriptæ, erit area FSG: ar. GSH $= \alpha : \beta$. Ducatur corda FH, ſecans SG in O, & ſequenti modo ſagitta GO definietur. Quoniam curvatura orbitæ FGH proficiſcitur a vi centripeta, ponamus vim centripetam qua cometa ſeu planeta ſpatium FGH peragrans ad ſolem S ſollicitatum, eſſe conſtantem $= P$; quia enim loca F & H ſatis ſibi ſunt vicina, hæc hypotheſis a veritate ſenſibiliter non recedit; Quo minus autem diſcrepet, ponamus P eam eſſe vim centripetam, quæ in loco medio G exercetur, ejusque directionem rectæ SG eſſe parallelam. Simili modo ſit vis centripeta terram in orbita circulari retinens $= p$, quæ quidem per ſe erit conſtans, at ponamus pariter, terram ab ea dum ſpatium $f g h$ percurrit, conſtanter in directione $g s$ ſollicitari. In F & f ducantur tangentes FMN, $f m n$, & ex H & h ipſis SG & $s g$ parallelæ HN & $h n$, erunt hæc intervalla HN & $h n$

effectus

effectus ab iſtis viribus centripetis producti: & quia hi effectus eodem tempore $= \alpha + \beta$ producuntur, erunt ii ipſis viribus proportionales, quare erit HN : $bn = P : p$. Erunt porro ex natura motus æquabilis quo tangentes FN & fn deſcriberentur, ſi nulla adeſſet vis ſollicitans, ſpatia FM: MN $= \alpha : \beta$; & $fm : mn = \alpha : \beta$, unde ob triangula FOM & FHN ſimilia erit quoque FO: OH $= \alpha : \beta$. Cum igitur & corda FH & ratio $\alpha : \beta$ detur ad locum medium G inveniendum ſecetur corda FH in O, ut ſit FO: OH $= \alpha : \beta$, & radius ex S per O productus tranſibit per locum G quæſitum. Supereſt ergo ut ſagitta GO definiatur, quod ex effectu virium ita præſtabitur: Erit nempe HN: GM $=$ FN^2: FM^2 $= (\alpha + \beta)^2 : \alpha^2$; & pari modo $bn : gm = fn^2 : fm^2 = (\alpha + \beta)^2 : \alpha^2$, unde fit HN: GM $= bn : gm$ ſeu HN: bn = GM: gm: eſt vero quoque HN: bn = OM: om; unde conducitur HN: bn = OG: og = P: p; ideoque erit $OG = \frac{P}{p} og$. Cum igitur ſit $og = \frac{2c \,\text{ſin}\, \alpha\tau, \,\text{ſin}\, \beta\tau}{\text{coſ}(\beta - \alpha)\tau}$; erit ſagitta quæſita $OG = \frac{2Pc \,\text{ſin}\, \alpha\tau . \,\text{ſin}\, \beta\tau}{p \,\text{coſ}\, (\beta - \alpha)\tau}$. Diviſa ergo corda FH in ratione temporum $\alpha : \beta$ in O, ductaque SO ſi capiatur $OG = \frac{2Pc \,\text{ſin}\, \alpha\tau, \text{ſin}\, \beta\tau}{p \,\text{coſ}((\beta - \alpha)\tau}$ erit G locus planetæ ſeu cometæ quæſitus, ſiquidem loca extrema F & H non multum ſint a ſe invicem remota. Cum autem vires centripetæ ſint in ratione reciproca duplicata diſtantiarum a ſole, erit

$P:p = cc;\ SG^2$: vel potius, quia vim ſumi oportet mediam inter extremas, erit ad veritatem accommodatius $P:p = 4cc:(SF + SH)^2$. Hac virium centripetarum ratione in computum ducta erit ſagitta $OG = \frac{8c^3 \sin \alpha\tau . \sin \beta\tau}{(SF+SH)^2 \cos(\beta-\alpha)\tau}$.

Q. E. J.

Corol. 1.

44. Si ratio temporum $\alpha:\beta$, atque adeo ratio ſegmentorum cordæ $FO:OH$ non multum a ratione æqualitatis diſcrepet, tum ſine errore loco $\frac{FS+SH}{2}$ ſcribi poterit SG ita ut ſit $OG = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta-\alpha)\tau}$. Ac ſi ratio $\alpha:\beta$ fuerit penitus ratio æqualitatis erit $OG = \frac{2c^3 \sin(\alpha\tau)^2}{SG^2} = \frac{c^3 \int v. 2\alpha\tau}{SG^2}$.

Coroll. 2.

45. Si ergo viciſſim detur locus G, in rectis SF & SH, quæ repræſentant loca heliocentrica, puncta F & H aſſignari poterunt, in quibus planeta ſeu cometa tempore α dier. antequam ad G, & tempore β dier. poſtquam in G fuerat, verſatur. Nam ex puncto G, ſumatur $GO = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 . \cos(\beta-\alpha)\tau}$ & per punctum O agatur recta FOH, ita ut ſit $FO:HO = \alpha:\beta$, erunt F & H loca quæſita.

Coroll. 3.

47. Quo minora ergo fuerint temporum intervalla α & β & quo propius ad rationem æqualitatis accedant, eo minus

nus hæc determinatio a veritate aberrabit. Neque vero plerumque aberratio a veritate fit ſenſibilis, niſi angulus FSH ſit ſatis magnus & circiter 10 vel 15 gradus ſuperet, quæ latitudo ſatis eſt magna, ut hinc utiles methodi ad orbitas tam planetarum quam cometarum deriventur.

Problema. XI.

47. Datis planetæ ſeu cometæ tribus locis geocentricis a ſe invicem non nimis remotis, una cum ejus diſtantia vera a terra tempore obſervationis mediæ; definire ejus planetæ ſeu cometæ orbitam veram, in qua circa ſolem movetur.

Solutio.

Tempore primæ obſervationis ſit terra in f exiſtente ſole in S, ſitque longitudo ſolis $= f$. In obſervatione ſecunda ſit terra in g, & longitudo ſolis $= g$; in obſervatione tertia ſit terra in H, & longitudo ſolis $= h$. Tempus autem inter primam & ſecundam obſervationem ſit $= \alpha$; & tempus inter ſecundam & tertiam obſervationem $= \beta$. Erit ergo angulus $f\,S\,g = g - f$, & angulus $g\,S\,h = h - g$: atque per theoriam ſolis dabuntur diſtantiæ S f, Sg, & S h. Repræſentet ſcilicet tabula planum eclipticæ, ſitque in prima obſervatione longitudo cometæ (ſeu planetæ) obſervata $=$ F, quam recta $f\,\zeta$ repræſentet: in obſervatione ſecunda ſit cometæ longitudo obſervata $=$ G, per rectam $g\,\eta$ repræſentata; & in obſervatione tertia ſit longitudo cometæ $=$ H, quam recta $h\,\theta$ exhibeat. Denique in obſervatione prima ſit Fig. 5.

ſit

latitudo cometæ $= \zeta$, in ſecunda $= \eta$, in tertia $= \theta$. Ex longitudinibus ergo obſervatis erunt anguli

$$S f \zeta = F - f, \quad S r \eta = G - f; \quad fmg = G - F$$
$$S g \eta = G - g; \quad S p \zeta = F - g; \quad gnh = H - G$$
$$S h \theta = H - h; \quad S s \theta = H - g;$$
$$S q y = G - h;$$

ducatur fx ipſi gm & $g\eta$ ipſi hn parallela, erit ob datos in triangulo Sfx omnes angulos cum latere Sf;

$$Sx = \frac{\text{ſin}(G-f)}{\text{ſin}(G-g)} Sf; \; fx = \frac{\text{ſin}(g-f)}{\text{ſin}(G-g)} Sf; \; \& \; gx = \frac{\text{ſin}(G-f)}{\text{ſin}(G-g)}$$

$Sf - Sg$. Deinde ob datos in triangulo fpx angulos cum latere Sx erit $fp = \frac{\text{ſin}(G-g)}{\text{ſin}(F-g)} fx = \frac{\text{ſin}(g-f)}{\text{ſin}(F-g)} Sf$; $px = \frac{\text{ſin}(G-F)}{\text{ſin}(F-g)} fx$, hinc eſt $fx : fp = \text{ſin}(G-F) : \text{ſin}(G-g) = gx : fm$, unde obtinetur $fm = \frac{\text{ſin}(G-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(G-g)}{\text{ſin}(G-F)} Sg$.

Tum erit $pm = \frac{\text{ſin}(g-f)}{\text{ſin}(F-g)} Sf + fm$. Ergo ſin $(G-g) : pm =$ ſin $(F-g) : gm$ unde fit $gm = \frac{\text{ſin}(F-g)}{\text{ſin}(G-g)} fm + \frac{\text{ſin}(g-f)}{\text{ſin}(G-g)} Sf$ ſeu

$$gm = \frac{\text{ſin}(G-f)\,\text{ſin}(F-g) + \text{ſin}(g-f)\,\text{ſin}(G-F)}{\text{ſin}(G-g)\;\text{ſin}(G-F)} Sf - \frac{\text{ſin}(F-g)}{\text{ſin}(G-F)} Sg.$$

at eſt ſin $(G-f)$ ſin$(F-g)$ + ſin $(g-f)$ ſin$(G-F)$ = ſin $(F-f)$ ſin$(G-g)$

ergo $gm = \frac{\text{ſin}(F-f)}{\text{ſin}(G-F)} Sf - \frac{\text{ſin}(F-g)}{\text{ſin}(G-F)} Sg$. Simili autem modo ratiocinando reperiuntur lineæ gn & hn eritque

$fm =$

$$f\,m = \frac{\text{ſin}\,(G-f)}{\text{ſin}\,(G-F)}\,Sf - \frac{\text{ſin}\,(G-g)}{\text{ſin}\,(G-F)}\,S\,g$$

$$g\,m = \frac{\text{ſin}\,(F-f)}{\text{ſin}\,(G-F)}\,Sf - \frac{\text{ſin}\,(F-g)}{\text{ſin}\,(G-F)}\,S\,g$$

$$g\,n = \frac{\text{ſin}\,(H-g)}{\text{ſin}\,(H-G)}\,Sg - \frac{\text{ſin}\,(H-h)}{\text{ſin}\,(H-G)}\,S\,h$$

$$h\,n = \frac{\text{ſin}\,(G-g)}{\text{ſin}\,(H-G)}\,S\,g - \frac{\text{ſin}\,(G-h)}{\text{ſin}\,(H-G)}\,S\,h.$$

Inventis jam punctis m & n, quibus ad ſequentem calculum erit opus, ſit G locus cometæ verus in ſecunda obſervatione, unde perpendicularis demittatur in eclipticam $G\eta$: ac ponatur ejus a terra diſtantia $= r$, erit ob latitudinem obſervatam $= \eta$, intervallum $g\,\eta = r$ coſ η; & $G\,\eta = r$ ſin η, dabitur ergo locus cometæ ad eclipticam relatus η, hincque lineæ $m\,\eta = g\,\eta - g\,m$; & $n\,\eta = g\eta - g\,n$. Ex η ducatur recta ad ſolem $S\,\eta$, atque demiſſo ex S in $g\,\eta$ perpendiculo SM, erit $SM = Sg$ ſin $(G-g)$ & $g\,M = Sg$ coſ $(G-g)$; hinc erit $M\eta = g\eta - g\,M$, & tang $\eta SM = \frac{\eta M}{SM} =$ cot $S\eta M$; atque $S\eta = \frac{SM}{\text{ſin}\,S\eta M}$. Deinde reperietur latitudo heliocentrica, erit enim tang $GS\eta = \frac{G\eta}{S\eta}$ & diſtantia a ſole $GS = \frac{G\eta}{\text{ſin}\,GS\eta} = \frac{S\eta}{\text{coſ}\,GS\eta}$. Sint nunc F & H loca cometæ in prima & tertia obſervatione, unde perpendicula in planum eclipticæ demiſſa cadere debent in rectas $f\,\zeta$ & $h\,\theta$, eritque

ducta corda FH, a radio SG in O ſecto, FO: HO $= \alpha :$ β, & erit ſagitta GO $= \frac{2c^3 \text{ ſin } \alpha\tau. \text{ ſin } \beta\tau}{SG^2 \text{coſ}(\beta-\alpha)\tau}$ (44). Demittatur quoque ex O perpendiculum Oo in planum eclipticæ, quod incidet in rectam $S\eta$; eritque $\eta o : GO = S\eta : SG$; ideoque $\eta o = GO$ coſ $GS\eta$. Cum igitur ſit $\zeta o \theta$ corda projectionis orbitæ in ecliptica, ob $\theta o : \zeta o = HO : FO = \beta : \alpha$, per punctum o duci debet recta $\zeta o \theta$ intra crura $m\zeta$ & $n\theta$, ita ut ſit $\zeta o : \theta o = \alpha : \beta$, hoc eſt in ratione data. Ad hoc efficiendum in $S\eta$ producta capiatur oi ut ſit $lo : oi = \alpha : \beta$, & per i ducatur ipſi $m\zeta$ parallela $i\theta$, cujus cum $n\theta$ interſectio θ dabit poſitionem cordæ $\zeta o \theta$ quæſitam. Sequenti autem modo per calculum puncta ζ & θ determinabuntur. Sit ang. $S\eta M = \mu$, qui cum detur erit in triangulo $m\eta l$, ſin $(G-F+\mu) : m\eta =$ ſin $(G-F) : \eta l =$ ſin $\mu : ml$ erit ergo

$$\eta l = \frac{\text{ſin }(G-F)}{\text{ſin }(G-F+\mu)} m\eta \text{ \& } ml = \frac{\text{ſin } \mu}{\text{ſin }(G-F+\mu)} m\eta$$ hinc

ob $lo = \eta l - \eta o$ dabitur lo: eritque $oi = \frac{\beta}{\alpha} lo$ & li

$= \frac{(\alpha+\beta)}{\alpha} lo = \lambda\theta$, ducta $\lambda\theta$ parallela ipſi li. Nunc conſideretur triangulum $\theta k \lambda$, eritque ſin $(H-F) : \lambda\theta =$ ſin $(G-F+\mu) : k\theta =$ ſin $(\mu-H+G) : k\lambda$; unde fit

$$k\theta = \frac{\text{ſin }(\mu+G-F)}{\text{ſin }(H-F)} \lambda\theta; \text{ \& } k\lambda = \frac{\text{ſin }(\mu-H+G)}{\text{ſin }(H-F)} \lambda\theta;$$ ſicque

cognoſcun-

cognoſcentur puncta θ & λ; ob $km = \frac{\text{ſin }(H-G)}{\text{ſin }(H-F)} mn$ & $nk = \frac{\text{ſin }(G-F)}{\text{ſin }(H-F)} mn$; eritque $l\lambda = ml + km - k\lambda$. Deinde fiat $\beta : \alpha = l\lambda : l\zeta$, eritque $l\zeta = \frac{\alpha}{\beta} l\lambda$, ideoque & punctum ζ eſt repertum. Quia ergo in triangulo $\zeta k \vartheta$ dantur latera $k\zeta$ & $k\theta$ cum angulo $\zeta k\theta = H - F$, reperietur latus $\zeta\theta$ cum angulo $k\zeta\theta$, unde erit angulus $So\zeta = 180° - k\zeta\theta - \mu - G + F$; ergo & latus $\zeta\theta$ & ejus poſitio erit cognita. Tum vero ex latitudinibus obſervatis erit $F\zeta = f\zeta$ tang ζ, & $H\theta = b\vartheta$ tang ϑ; ex quo & cordæ FH poſitio innoteſcit. Producatur corda HF, donec in plano eclipticæ cum $\vartheta\zeta$ in N occurrat, eritque recta SN interſectio orbitæ cometæ & eclipticæ, ideoque linea nodorum; & in N quidem erit nodus aſcendens, ſi latitudines obſervatæ fuerint boreales, & loca cometæ ita ſint diſpoſita, ut figura repræſentat. Ad punctum N inveniendum, fiat $H\vartheta - F\zeta : \zeta\vartheta = H\vartheta : \vartheta N$, unde fit $\vartheta N = \frac{\zeta\vartheta}{H\vartheta - F\zeta} . H\vartheta$; & $\frac{H\vartheta - F\zeta}{\zeta\vartheta}$ dabit anguli $HN\vartheta$ tangentem. Deinde in triangulo SoN ob data latera So & No cum angulo $SoN = So\zeta$, reperientur anguli NSo, SNo, cum latere SN. Hinc primum ſi angulus $NS\eta + \eta Sg$ a longitudine terræ in obſervatione media, quæ eſt $= 6^s + g$, ſubtrahatur, remanebit longitudo heliocentrica nodi N. Tum ex o in SN demittatur perpendiculum oP, ductaque recta OP angulus OPo monſtrabit in-

clinationem orbitæ cometæ ad eclipticam, Erit vero $oP^{\bullet} = No.\sin SNo$; & $Oo = No.\tang HN\vartheta$, unde erit $\tang OPo = \frac{\tang HN\vartheta}{\sin SNo} = \frac{H\vartheta - F\zeta}{\zeta\vartheta.\sin SNo}$. Deinde erit $\cos SNH = \frac{NP}{NO} = \frac{NP}{No} \cdot \frac{No}{NO} = \cos SNo. \cos HN\vartheta$. Denique invenitur $NF = \frac{F\zeta}{\sin HN\vartheta}$; & $HN = \frac{H\vartheta}{\sin HN\vartheta}$, atque hinc in plano orbitæ cometæ SNFH dabuntur latus SN; angulus SNH, & latera NF & NH, unde reperientur anguli NSF & NSH cum lateribus SF & SH; ideoque dabuntur duæ cometæ a ſole diſtantiæ FS & HS, una cum angulo FSH & tempore $= \alpha + \beta$ dier. quo cometa ex F in H pervenit, ex quibus orbita cometæ invenietur per probl. V. Q. E. I.

Coroll. 1.

48. Quo igitur hæc orbitæ cometæ vel planetæ determinatio magis ſit exacta; neceſſe eſt ut obſervationes non nimis à ſe invicem diſtent, atque ut ratio inter tempora $\alpha : \beta$ proxime ſit ratio æqualitatis. Tum vero hoc maxime requiritur, ut obſervationes ſumma cura ſint inſtitutæ atque ut diſtantia cometæ a terra in obſervatione media a veritate quam minimum aberret.

Coroll. 2.

48. Quando longitudo cometæ in obſervatione media $g\eta$ in ipſam longitudinem ſolis gS incidit, tum angulus $S\eta g$

ſit vel

fit vel nullus vel 180°, hocque adeo caſu calculus non mediocriter contrahitur.

Scholion I.

50. Operatio hæc vel per accuratam deſcriptionem figuræ ſuper charta ſatis magna inſtitui poteſt, vel, quod magis eſt ſuadendum, per calculum trigonometricum; quemadmodum ergo hunc calculum quam breviſſime inſtitui oporteat prætermiſſo omni ratiocinio, hic ob oculos ponamus, quo commodius eo ad orbitas cometarum inveſtigandas uti liceat. Notentur igitur primum quæ ſunt per obſervationes data:

Tempus elapſum ab Obſervatione I ad II $=$ α dieb.
Tempus elapſum ab Obſervatione II ad III $=$ β dieb.

Obſervat.	Long. Solis	Diſt. ⊙ a ♁	Long. Com.	Lat. Com.
I	f	Sf	F	ζ
II	g	Sg	G	η
III	h	Sh	H	θ

Hinc definiantur anguli $fmg = \zeta m \eta = G - F$
$gnh = \eta n \theta = H - G$
$fkh = \zeta k \theta = H - F$

itemque

$Sf\zeta = F - f$; $Sr\eta = G - f$; $Ss\theta = H - g$
$Sg\eta = G - g$; $Sp\zeta = F - g$; $Sq\eta = G - h$
$Sh\theta = H - h$; Ex his reperitur

$$fm = \frac{\sin Sr\eta}{\sin fmg} Sf - \frac{\sin Sg\eta}{\sin fmg} Sg$$

$$gm = \frac{\sin Sf\zeta}{\sin fmg} Sf - \frac{\sin Sp\zeta}{\sin fmg} Sg$$

 $gn =$

$$g\,n = \frac{\text{ſin } S\,s\,\theta}{\text{ſin } g\,n\,b}\,S\,g - \frac{\text{ſin } S\,b\,\theta}{\text{ſin } g\,n\,b}\,S\,b$$

$$b\,n = \frac{\text{ſin } S\,g\,\eta}{\text{ſin } g\,n\,b}\,S\,g - \frac{\text{ſin } S\,q\,\eta}{\text{ſin } g\,n\,b}\,S\,b$$

unde $mn = gm - gn$; & $mk = \frac{\text{ſin}\,g\,n\,b}{\text{ſin}\,f\,k\,b}\,mn$; $nk = \frac{\text{ſin}\,f\,m\,g}{\text{ſin}\,f\,k\,b}\,mn$

Sit in obſervatione II diſtantia cometæ a terra $= r$ erit $G\eta = r \text{ ſin } \eta$; $g\eta = r \text{ coſ}\eta$; $m\eta = g\eta - gm$; $n\eta = g\eta - gn$.

Porro eſt tang $\frac{1}{2}(g\eta S - g S\eta) = \frac{gS - g\eta}{gS + g\eta} \cot \frac{1}{2} Sg\eta$: unde erit $g\eta S = \frac{1}{2}(g\eta S - gS\eta) + 90 - \frac{1}{2} Sg\eta$; & $gS\eta = 90 - \frac{1}{2}Sg\eta - \frac{1}{2}(g\eta S - gS\eta)$

atque $S\,\eta = \frac{\text{ſin } S\,g\,\eta}{\text{ſin } g\,\eta\,S}\,S\,g$.

Deinde eſt tang $GS\eta = \frac{G\,\eta}{S\,\eta}$; & $SG = \frac{G\,\eta}{\text{ſin } GS\eta}$

Ponatur angulus $1774_{10}\,{}^{98}_{08}{}'' = \tau$, ut ſit $l\tau = 3,2489776$ & quærantur anguli $\alpha\tau$ & $\beta\tau$; ſitque $c = 100000$; erit $GO = \frac{2c^3 \text{ ſin } \alpha\tau.\ \text{ſin } \beta\tau}{SG^2 \text{ coſ}(\beta-\alpha)\tau}$; & $\eta o = GO \text{ coſ } GS\eta$; $So = S\eta - o\eta$

Deinde eſt ang. $\zeta lo = g\eta S - \zeta m\eta$; & $l\eta = \frac{\text{ſin } \zeta\,m\,\eta}{\text{ſin } \zeta\,l\,o}\,m\,\eta$

& $ml = \frac{\text{ſin}\,g\,\eta\,S}{\text{ſin}\,\zeta\,l\,o}\,m\eta$: indeque $lo = l\eta - \eta o$, ex hisque

$\lambda\theta = \frac{\alpha + \beta}{\alpha}\,lo$; $k\theta = \frac{\text{ſin}\,\zeta\,l\,o}{\text{ſin}\,\zeta\,k\,\theta}\,\lambda\vartheta$; & (ob $k\vartheta\lambda = \zeta lo - \zeta k\vartheta$)

$k\lambda =$

$k\lambda = \frac{\text{ſin } k\vartheta\lambda}{\text{ſin } \zeta k\vartheta} \lambda\vartheta$, unde $l\lambda = ml + km - k\lambda$, $l\zeta = \frac{\alpha}{\beta} l\lambda$ & $k\zeta = km + ml + l\zeta$, atque tang $\frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta) = \frac{k\zeta - k\vartheta}{k\zeta + k\vartheta}$ cot $\frac{1}{2}\zeta k\vartheta$, & $k\vartheta\zeta = 90 - \frac{1}{2}\zeta k\vartheta + \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$; $k\zeta\vartheta = 90 - \frac{1}{2}\zeta k\vartheta - \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$ atque $\vartheta\zeta = \frac{\text{ſin } \zeta k\vartheta}{\text{ſin } k\zeta\vartheta} k\vartheta$; unde $S o \zeta = 190° k\zeta\vartheta - \zeta l o$. His inventis erit $f\zeta = fm + ml + l\zeta$: $h\vartheta = hn + nh + k\vartheta$; hinque $F\zeta = f\zeta$ tang ζ & $H\vartheta = h\vartheta$ tang ϑ, ac porro tang $HN\vartheta = \frac{H\vartheta - F\zeta}{\zeta\vartheta}$ & $\vartheta N = \frac{H\vartheta}{\text{tang } HN\vartheta}$; ob $\vartheta o = \frac{\beta}{\alpha + \beta}\zeta\vartheta$, erit $No = N\theta - \theta o$. Deinceps eſt tang $\frac{1}{2}(SNo - oSN) = \frac{oS - oN}{oS + oN}$ cot $\frac{1}{2} So\zeta$ & $SNo = 90 - \frac{1}{2}So\zeta + \frac{1}{2}(SNo - oSN)$; $oSN = 90 - \frac{1}{2}So\zeta - \frac{1}{2}(SNo - oSN)$ & $SN = \frac{\text{ſin } So\zeta}{\text{ſin } oSN} No$, unde longitudo heliocentrica nodi $N = 6' + g - gS\eta - oSN$, & inclinationis orbitæ ad Eclipticam tang $= \frac{\text{tang } HN\theta}{\text{ſin } SNo}$. Deinde eſt coſ $SNH =$ coſ SNo. coſ $HN\vartheta$ & $FN = \frac{F\zeta}{\text{ſin } HN\theta}$; $HN = \frac{H\theta}{\text{ſin } HN\theta}$. Denique ſiet tang $\frac{1}{2}(NFS - NSF) = \frac{SN - NF}{SN + NF}$ cot $\frac{1}{2} SNH$; $NFS = 90 - \frac{1}{2} SNH + \frac{1}{2}(NFS - NSF)$ $NSF = 90 - \frac{1}{2} SNH - \frac{1}{2}(NFS - NSF)$ & $SF = \frac{\text{ſin } SNH}{\text{ſin } NFS} SN$; &

tang

$\text{tang}\,\frac{1}{2}\,(NHS-NSH) = \frac{SN-NH}{SN+NH}\cot\frac{1}{2}\,SNH$; unde $NHS = 90 - \frac{1}{2}SNH + \frac{1}{2}(NHS-NSH)$ & $NSH = 90 - \frac{1}{2}SNH - \frac{1}{2}(NHS-NSH)$, & $SH = \frac{\text{ſin}\ SNH}{\text{ſin}\ NHS}SN$ atque $FSH = NSH - NSF$.

Scholion. 2.

51. Inventis duabus diſtantiis FS & HS cum angulo FSH & tempore inter hæc loca elapſo $= \alpha + \beta$ dier. orbita ſequenti calculo definietur. Sit

diſtantia $FS = y$, tempus $\alpha + \beta = T$

diſtantia $HS = z$, & ang $FSH = \phi$

ob $m = 271989,735$ & $lm = 5,4345525139$ erit orbitæ latus rectum $b = \left(\frac{yyzz}{4m^2T^2} + \frac{1}{3}\sqrt{yz}\right)(\text{ſin}\ \phi)^2$. Sit diſtantia perihelii a ſole $= a$; & anomalia vera loci F ſit $= v$, erit $\text{tang}\ v = \cot\phi - \frac{(b-z)y}{(b-y)z\ \text{ſin}\ \phi}$, & $a = \frac{by\ \text{coſ}\ v}{b-y+y\ \text{coſ}\ v}$. Sumatur $\text{tang}\ \frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}\ \text{tang}\ \frac{1}{2}v$, ſi quidem fuerit $2a > b$ & orbita ellipſis: eritque tempus, quo cometa a perihelio ad locum F pervenit, in diebus expreſſum $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\cdot\left(\omega - \frac{(b-a)}{a}\ \text{ſin}\ \omega\right)$, ſin autem fuerit $b > 2a$ & curva hyperbola, ſumatur $\text{tang}\ \frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}\ \text{tang}\ \frac{1}{2}v$, & erit tempus

quo

quo cometa a perihelio ad F pervenit in diebus $= \frac{a^3}{2m(b-a)^{\frac{3}{2}}}$ $\left(\frac{b-a}{a} \text{tang}\, \omega - l\, \text{tang}\, (45^\circ + \tfrac{1}{2}\omega)\right)$. At ſi orbita cometæ fuerit vel parabola vel ad eam proxime accedat, ponatur tang $\frac{1}{2} v = t$; & $\delta = 2a - b$ & $n = \frac{\delta}{b} = \frac{2a-b}{b}$, eritque tempus, quo cometa a perihelio ad F pervenit, in diebus expreſſum $=$

$$\frac{aa}{m\sqrt{b}} \left(t + \tfrac{1}{3} t^3 \begin{matrix} -\tfrac{2}{5} n \\ +\tfrac{3}{5} nn \end{matrix} t^5 \begin{matrix} +\tfrac{3}{7} nn \\ -\tfrac{4}{7} n^3 \end{matrix} t^7 \begin{matrix} -\tfrac{4}{9} n^3 \\ +\tfrac{5}{9} n^4 \end{matrix} t^9 \ \&c. \right)$$

Cum igitur tempus conſtet, quo cometa in F eſt verſatus, ſimul temporis momentum habebitur, quo cometa per perihelium tranſiit. Denique ſi ab anomalia vera $= v$ loci F ſubtrahatur angulus FSN, habebitur diſtantia nodi N a perihelio. Ceterum hoc problema parum utilitatis habere videatur, cum nunquam parallaxis cometæ tam exacte definiri queat, quam ad hoc inſtitutum opus eſt; at vero ſummus uſus hujus reſolutionis perſpicietur in problemate ſequenti.

Problema XII.

52. Ex aliquot obſervationibus cometæ ejus orbitam veram determinare.

Solutio.

Ex obſervationibus cometæ, quibus vel ejus diſtantiæ a ſtellis fixis, vel altitudines meridianæ ſunt menſuratæ, elicien-

tur ejus longitudines ac latitudines geocentricæ, & ad unamquamque obſervationem notetur temporis momentum, quo eſt facta, ſecundum tempus medium. Ex his cometæ locis in ordinem diſpoſitis eligantur tria non nimis a ſe invicem remota, ita ut media ratione temporis fere medium inter jaceat inter extremas. Tum pro lubitu in obſervatione media fingatur cometæ diſtantia a terra, ac per problema præcedens ex his tribus locis & diſtantia ficta determinetur orbita cometæ, quæ erit vera, ſi diſtantia illa ficta cum vera conveniat, contra autem falſa. Sumatur ergo obſervatio quædam quarta a tribus illis electis longiſſime remota, atque ex orbita eruta ad ejus tempus computetur locus cometæ geocentricus, qui ſi cum obſervato congruat, docebit orbitam erutam eſſe veram, ſin minus congruat, eſſe falſam. Fingantur ergo ſimul duæ tresve diſtantiæ, quas cometa in obſervatione media habuerit, inter ſe diverſæ, & ex qualibet definiatur orbita cometæ, & diſpiciatur quantum quæque ab obſervatione quarta diſcrepet. Atque hinc etiamſi nulla orbita hoc modo inventa ſit vera, tamen facile intelligetur, quænam ad veritatem propius accedat; atque adeo, ſi diſſenſus non fuerit admodum magnus, per interpolationem vera cometæ orbita erui poterit. Sin autem diſſenſus ſit adhuc nimis magnus, tum ſaltem diſtantia cometæ in obſervatione media propius cognoſcetur; quare fingendis aliquot

novis

novis diſtantiis a veritate minus abhorrentibus, ſi ex iis pari modo orbitæ definiantur, & cum obſervatione quarta conferantur, per interpolationem ſatis exacte vera orbita cometæ concludi poterit. Q. E. J.

Coroll. 1.

53. Quoniam prima fictio ideo tantum fit, ut orbita cometæ veræ tantum propinqua obtineatur, non opus eſt, ut calculus omni cura inſtituatur, ſed ſufficiet per conſtructionem geometricam ex diſtantiis fictis orbitam eruere.

Coroll. 2.

54. Eodem modo poſſent etiam planetarum orbitæ investigari, atque ex quatuor obſervationibus definiri. At quoniam in planetis per continuas obſervationes tempora periodica & lineæ nodorum facile determinantur, his cognitis certior aperitur via ad eorum orbitas definiendas, quam potius ſequi convenit.

Scholium.

55. Hujus methodi utilitas aptiſſime per exemplum docebitur, ſimulque patebit, quemadmodum commodiſſime calculum ſatis prolixum inſtitui oporteat. Hunc in finem cometam, qui circa finem A. 1680 & initium ſequentis anni

apparuit, potiſſimum inveſtigari conveniet, quippe quo vix ullus alius majori cura atque opportunitate non ſolum eſt obſervatus, ſed etiam ejus orbita per calculum determinata. Cum igitur hujus cometæ & obſervationes per quadrimeſtre fere temporis ſpatium habeantur accuratiſſimæ, & ipſa ejus orbita Virorum Celeberrimorum Neutoni & Halleji ſtudiis ſit determinata, etiamſi actum agere videar, tamen hoc exemplo mea methodus non ſolum maxime illuſtrabitur, ſed etiam ex conſenſu fortiſſime confirmabitur. Obſervationes autem hujus cometæ in Principiis Math. Philoſophiæ Neutonianæ recenſentur, unde eas hic depromam.

Cometæ, qui circa finem anni 1680. & initium anni 1681. apparuit, loca obſervata ad tempus medium ſtili veteris & meridianum Londinenſem reducta.

A.1680.M:Nov.	*d. h. m.*	Longitudo	Latitudo
	16,17,10	6ˢ,8°,0′	0°,44,′ auſtr.
	17,17,10	6,12,52	1, 0
	18,21,44	6,18,40	1, 18
	19,17,10	6,22,48	1, 30
	20,17, —	6,27,52	1, 45
	21,17, —	7, 2,56	1, 58
	23,17,15	7,12,58	2, 20
	24,17,30	7,17,53	2, 29
M.Dec.	12 4,46,0″	9, 6,31,21″	8,26,0″ bor.
	21, 6,36,59	10, 5, 7,38	21,45,30
	24, 6,17,52	10,18,49,10	25,23,24
	26, 5,20,44	10,28,24, 6	27, 0,57
	29, 8, 3, 2	11,13,11,45	28,10, 5
	30, 8,10,26	11,17,39, 5	28,11,12
A.1681.M. Jan.	5, 6, 1,38	0, 8,49,10	26,15,26
	9, 7, 0,53	0,18,43,18	24,12,42
	10, 6, 6,10	0,20,40,57	23,44, 0
	13, 7, 8,55	0,25,59,34	22,17,36
	25, 7,58,42	1, 9,35,48	17,56,54
	30, 8,21,53	1,13,19,36	16,40,57
M.Febr.	2, 6,34,51	1,15,13,48	16, 2, 2
	5, 7, 4,41	1,16,59,52	15,27,23
	25, 8,41, —	1,26,18,17	12,46,54
	27; 8,26, —	1,27, 4,24	12,36,12
M.Mart.	1,11,10, —	1,27,53, 6	12,24,52
	2, 8,10, —	1,28,12,27	12,20, 0
	5,11,39, —	1,29,20,51	12, 3,30
	9, 8,38, —	2, 0,43, 3	11,45,53

Inveſtigatio Orbitæ hujus Cometæ.

Eligantur tres ſequentes obſervationes Menſe Januario A. 1681. factæ

Tempus	Long. Solis	Diſt. Sol. a Ter.	Long. Comet.	Lat. Cometæ
$5^d, 6^h, 1', 38''$	$9^s, 26^\circ, 22', 18''$	9 8 3 6 3, 5	$0^s, 8^\circ, 49', 10''$	$26^\circ, 15', 26''$
9, 7, 0, 53	10, 0, 29, 2	9 8 4 0 7, 0	0, 18, 43, 18	24, 12, 42
13, 7, 8, 55	10, 4, 33, 20	9 8 4 5 8, 8	0, 25, 59, 34	22, 17, 36

Hinc erit

$$\alpha = 4^d, 0^h, 59', 15'' = 4,0411;\ l\alpha = 0,6064996$$

$$\beta = 4^d, 0^h, 8', 2'' = 4,0055;\ l\beta = 0,6026567$$

$$\alpha + \beta = 8,0466;\ \&\ l(\alpha + \beta) = 0,9056124$$

$$f = 9^s, 26^\circ, 22', 18'';\ F = 0^s, 8^\circ, 49', 10'';\ \zeta = 26^\circ, 15', 26''$$

$$g = 10, 0, 29, 2;\ G = 0, 18, 43, 18;\ \eta = 24, 12, 42$$

$$h = 10, 4, 33, 20;\ H = 0, 25, 59, 34;\ \theta = 22, 17, 36$$

$$fmg = \zeta m \eta = G - F = 9^\circ, 54', 8''$$

$$gnh = \eta n \theta = H - G = 7^\circ, 16', 16''$$

$$fkh = \zeta k \theta = H - F = 17, 10, 24$$

$$Sf\zeta = F - f = 72, 26, 52$$

$$Sg\eta = G - g = 78, 14, 16$$

$$Sh\theta = H - h = 81, 26, 14$$

$$Sr\eta = G - f = 82, 21, 0$$

$$Sp\zeta = F - g = 68, 20, 8$$

$$Ss\theta = H - g = 85, 30, 32$$

$$Sq\eta = G - h = 74, 9, 58$$

$$Sf = 98363,5;\ lSf = 4,9928340$$

$$Sg = 98407,0;\ lSg = 4,9930260$$

$$Sh = 98458,8;\ lSh = 4,9932545$$

Jam ſequentes calculi inſtituantur.

l S f

$l\,Sf = 4,\ 9928340$

subt. $l\,\text{ſin}\,fmg = 9,\ 2354458$

$5,\ 7573882$

$l\,\text{ſin}\,Sr\eta = 9,\ 9961174$

$l\,\text{ſin}\,Sf\zeta = 9,\ 9792946$

$l\,\frac{\text{ſin}\,Sr\eta}{\text{ſin}\,fmg}\,Sf = 5,\ 7535056$

$l\,\frac{\text{ſin}\,Sf\zeta}{\text{ſin}\,fmg}\,Sf = 5,\ 7366828$

$l\,Sg = 4,\ 9930260$

$l\,\text{ſin}\,fmg = 9,\ 2354458$

$5,\ 7575802$

$l\,\text{ſin}\,Sg\eta = 9,\ 9907836$

$l\,\text{ſin}\,Sp\zeta = 9,\ 9681848$

$l\,\frac{\text{ſin}\,Sg\eta}{\text{ſin}\,fmg}\,Sg = 5,\ 7483638$

$l\,\frac{\text{ſin}\,Sp\zeta}{\text{ſin}\,fmg}\,Sg = 5,\ 7257650$

$\frac{\text{ſin}\,Sr\eta}{\text{ſin}\,fmg}\,Sf = 566898,\ 9$

$\frac{\text{ſin}\,Sg\eta}{\text{ſin}\,fmg}\,Sg = 560226,\ 8$

$fm = 6672,\ 1$

$\frac{\text{ſin}\,Sf\zeta}{\text{ſin}\,fmg}\,Sf = 545359,\ 4$

$\frac{\text{ſin}\,Sp\zeta}{\text{ſin}\,fmg}\,Sg = 531820,\ 4$

$gm = 13539,\ 0$

$gn = 5875,\ 0$

$mn = 7664,\ 0$

$l\,Sg = 4,\ 9930260$

$l\,\text{ſin}\,gnb = 9,\ 1023116$

$5,\ 8907144$

$l\,\text{ſin}\,Ss\theta = 9,\ 9986644$

$l\,\text{ſin}\,Sg\eta = 9,\ 9907836$

$l\,\frac{\text{ſin}\,Ss\theta}{\text{ſin}\,gnb}\,Sg = 5,\ 8893788$

$l\,\frac{\text{ſin}\,Sg\eta}{\text{ſin}\,gnb}\,Sg = 5,\ 8814980$

$l\,Sb = 4,\ 9932545$

$l\,\text{ſin}\,gnb = 9,\ 1023116$

$5,\ 8909429$

$l\,\text{ſin}\,Sb\theta = 9,\ 9951318$

$l\,\text{ſin}\,Sq\eta = 9,\ 9832008$

$l\,\frac{\text{ſin}\,Sb\theta}{\text{ſin}\,gnb}\,Sb = 5,\ 8860747$

$l\,\frac{\text{ſin}\,Sq\eta}{\text{ſin}\,gnb}\,Sb = 5,\ 8741437$

$\frac{\text{ſin}\,Ss\theta}{\text{ſin}\,gnb}\,Sg = 775137,\ 7$

$\frac{\text{ſin}\,Sb\theta}{\text{ſin}\,gnb}\,Sb = 769262,\ 7$

$gn = 5875,\ 0$

$\frac{\text{ſin}\,Sg\eta}{\text{ſin}\,gnb}\,Sg = 761198,\ 5$

$\frac{\text{ſin}\,Sq\eta}{\text{ſin}\,gnb}\,Sb = 748417,\ 0$

$bn = 12781,\ 5$

$l\,mn = 3,\ 8844555$

$l\,\text{ſin}\,fkb = 9,\ 4702096$

$4,\ 4142459$

m k

mk	=	3285, 168	l ſin gnb	=	9, 1023116
nk	=	4463, 666	l ſin fmg	=	9, 2354458
			$l\,mk$	=	3, 5165575
			$l\,nk$	=	3, 6496917

Ut jam labori in ſcrutando valore diſtantiæ $gG = r$ vero proximo parcamus, conſulamus Theoriam Neutoni, quæ pro hoc tempore exhibet diſtantiam cometæ a ſole = .1 1 0 9 7 0, unde elicitur diſtantia cometæ a terra = 7 2 7 4 7. Hanc obrem pro r aſſumam hos duos valores 7 2 7 0 0 & 7 2 8 0 0.

Sit ergo r	=	72700	72800
lr	=	4, 8615344	4, 8621314
add l ſin η	=	9, 6128990	9, 6128990
add l coſ η	=	9, 9600122	9, 9600122
$l\,G\eta$	=	4, 4744334	4, 4750304
$l\,g\eta$	=	4, 8215466	4, 8221436
$g\eta$	=	66305, 05	66396, 26
gm	=	13539, 0	13539, 0
gn	=	5875, 0	5875, 0
$m\eta$	=	52766, 05	52857, 26
$n\eta$	=	60430, 05	60521, 26
Sg	=	98407, 0	98407, 0
$g\eta$	=	66305, 05	66396, 26
$Sg + g\eta$	=	164712, 05	164803, 26
$Sg - g\eta$	=	32101, 95	32010, 74
$l(Sg - g\eta)$	=	4, 506531[illegible]	4, 5052957
$l(Sg + g\eta)$	=	5, 216725[illegible]	5, 2169658
		9, 2898060	9, 2883299
l tang $(90 - \frac{1}{2}Sg\eta)$	=	10, 089[illegible]	10, 0897890
l tang $\frac{1}{2}(g\eta S - gS\eta)$	=	9, 3795950	9, 3781189

$\frac{1}{2}(g\eta S - gS\eta)$

$Sg\eta = 78^\circ, 14', 16''$

$\frac{1}{2}Sg\eta = 39, 7', 8$

Compl. $= 50, 52', 52''$

$\frac{1}{2}(g\eta S - gS\eta)$ =	13°,28′,38″	13°,25′,59″
$90 - \frac{1}{2}Sg\eta$ =	50, 52, 52	50, 52, 52′
$g\eta S$ =	64, 21 30	64, 18, 51
$gS\eta$ =	37, 24, 14	37, 26, 53
Ad $l\,Sg$ =	4, 9930260	4, 9930260
add l ſin $Sg\eta$ =	9, 9907836	9, 9907836
	14, 9838096	14, 9838096
ſubtr. l ſin $g\eta S$ =	9, 9549744	9, 9548136
$l\,S\eta$ =	5, 0288352	5, 0289960
a $l\,G\eta$ =	4, 4744334	4, 4750304
l tang $GS\eta$ =	9, 4455982	9, 4460344
$GS\eta$ =	15°,35′,20″	15°,36′,14″
A $l\,G\eta$ =	4, 4744334	4, 4750304
ſubtr. l ſin $GS\eta$ =	9, 4293210	9, 4297284
$l\,SG$ =	5, 0451124	5, 0453020
$l\,\tau$ =	3, 2489776	
$l\,\alpha$ =	0, 6064996	
$l\,\beta$ =	0, 6026567	
$l\,\alpha\tau$ =	3, 8554772	
$l\,\beta\tau$ =	3, 8516343	
$\alpha\tau$ =	7169″,3 =	1°,59′,29″
$\beta\tau$ =	7106″,1 =	1°,58′,26″
$(\alpha-\beta)\tau$ =	63, 2 =	1′, 3″
l ſin $\alpha\tau$ =	8, 5409422	
l ſin $\beta\tau$ =	8, 5371103	
$l\,2c^3$ =	15, 3010300	
	32, 3790825	
ſubt. l coſ$(\alpha-\beta)\tau$ =	10, 0000000	
	12, 3790825	12, 3790825
ſubtr 2 $l\,SG$ =	10, 0902248	10, 0906040

lGO =	2, 2888577	2, 2884785
add. l coſ G S η =	9, 9837231	9, 9836924
$l\,\eta o$ =	2, 2725808	2, 2721709
ηo =	187,32	187,14
ab Sη =	106864,9	106904,5
So =	106677,6	106717,4
Ang. $g\eta$S =	64°,24′,30″	64°,18′,51″
$\zeta m\eta$ =	9, 54, 8	9, 54, 8
ζlo =	74, 15, 38	74, 12, 59″
$lm\eta$ =	4, 7223546	4, 7231046
ſubtr. l ſin ζlo =	9, 9834031	9, 9833086
	4, 7389515	4, 7397960
add l ſin $\zeta m\eta$ =	9, 2354458	9, 2354458
add l ſin $g\eta$S =	9, 9549744	9, 9548136
l $l\eta$ =	3, 9743973	3, 9752418
$l\,ml$ =	4, 6939259	4, 6946096
Ergo $l\eta$ =	9427, 52	9445, 87
ſubtr. ηo =	187, 32	187, 14
lo =	9240, 20	9258, 73
l lo =	3, 9656814	3, 9665515
add $l\,\frac{(\alpha+\beta)}{\alpha}$ =	0, 2991128	0, 2991128
erit $l\,\lambda\theta$ =	4, 2647942	4, 2656643
Ang. ζlo =	74°,15′,38′	74°,12′,59″
ſubtr. $\zeta k\theta$ =	17°,10, 24	17, 10, 24
$k\theta\lambda$ =	57 ,5 ,14	57 ,2 ,35
$l\lambda\theta$ =	4, 2647942	4, 2656643
ſubtr. l ſin $\zeta k\theta$ =	9, 4702096	9, 4702096
	4, 7945846	4, 79[illegible]4547
add l ſin ζlo =	9, 9834031	9, 9833086
add l ſin $k\theta\lambda$ =	9, 9240200	9, 9238032

$lk\theta$ =

$l\,k\theta =$	4, 7779877	4, 7787633
$l\,k\lambda =$	4, 7186046	4, 7192579
$m\,l =$	49422, 63	49500, 50
$k\,m =$	3285, 17	3285, 17
$k\,l =$	52707, 80	52785, 67
ſubtr. $k\,\lambda =$	52312, 40	52391, 15
$l\,\lambda =$	395, 40	394, 52
$l\,l\,\lambda =$	2, 5970367	2, 5960690
add. $l\,\frac{\alpha}{\beta} =$	0, 0038429	0, 0038429
$l.\,l\zeta =$	2, 6008796	2, 5999119
$\frac{1}{2}\,\zeta k\theta =$	8°,35′,12″	8°,35′,12″
$90 - \frac{1}{2}\,\zeta k\theta =$	81°,24′,48″	81°,24′,48″
$k\,\zeta =$	3285, 17	3285, 17
$m\,l =$	49422, 63	49500, 50
$l\,\zeta =$	398, 91	398, 02
$k\,\zeta =$	53106, 71	53183, 69
$k\,\theta =$	59977, 42	60084, 63
$k\theta + k\zeta =$	113084, 13	113268, 32
$k\theta - k\zeta =$	6870, 71	6900, 94
$l\,(k\theta - k\zeta) =$	3, 8370016	3, 8389082
$l\,(k\theta + k\zeta) =$	5, 0534016	5, 0541084
	8, 7836000	8, 7847998
l tang $(90 - \frac{1}{2}\zeta k\theta) =$	10, 8210294	10, 8210294
	9, 6046294	9, 6058292
$\frac{1}{2}\,(k\zeta\theta - k\theta\zeta) =$	21°,55′,7″	21°,58′,24″
$\frac{1}{2}\,(k\zeta\theta + k\theta\zeta) =$	81, 24, 48	81, 24, 48
$k\zeta\theta =$	103, 19,55	103, 23,12
$k\theta\zeta =$	59, 29,41	59, 26,24
$k\zeta\theta + \zeta l o =$	177, 35,33	177, 36,11
$S\,o\,\zeta =$	2°, 24,27	2, 23,49

A l ſin $\zeta k\theta$ =	9, 4702096	9, 4702096
ubtr. l ſin $k\zeta\theta$ =	9, 9881354	9, 9880368
	9, 4820742	9, 4821728
add $l\,k\theta$ =	4, 7779877	4, [illegible]87633
$l\,\theta\zeta$ =	4, 2600619	4, 26[illegible]361
fm =	6672, 1	6672, 1
ml =	49422, 6	49500, 5
$l\zeta$ =	398, 9	398, 0
erit $f\zeta$ =	56493, 6	56570, 6
bn =	12781, 5	12781, 5
nk =	4463, 6	4463, 6
$k\vartheta$ =	59977, 4	60084, 6
erit $b\vartheta$ =	77222, 6	77329, 8
ad $l\,f\zeta$ =	4, 7519993	4, 7525908
add $l$tang ζ =	9, 6931129	9, 6931129
lFζ =	4, 4451122	4, 4457037
$l\,b\vartheta$ =	4, 8877445	4, 8883470
add. l tang ϑ =	9, 6127775	9, 6127775
lHϑ =	4, 5005220	4, 5011245
Ergo Hϑ =	31660, 8	31704, 8
Fζ =	27868, 4	27906, 4
Hϑ−Fζ =	3792, 4	3798, 4
l (Hϑ − Fζ) =	3, 5789141	3, 5796007
ſubtr. $l\,\zeta\vartheta$ =	4, 2600619	4, 2609361
$l$tang HNϑ =	9, 3188522	9, 3186646
a l Hϑ =	4, 5005220	4, 5011245
l Nϑ =	5, 1816698	5, 1824599
Ang. HNϑ =	11°,46′,15″	1[illegible]°,45′,57″
Nϑ =	151939[illegible]2	1[illegible]2215, 9

$l\zeta\vartheta$ =

$l\zeta\theta$ =	4, 2600619	4, 2609361
ſubt. $l\frac{\alpha+\beta}{\beta}$ =	0, 3029557	0, 3029557
$l\theta o$ =	3, 9571062	3, 9579804
θo =	9059, 5	9077, 8
N o =	142879, 7	143138, 1
$\frac{1}{2}$ S $o\zeta$ =	1°,12′,13½″	1°,11′,54½″
90 − $\frac{1}{2}$ S $o\zeta$ =	88°,47′,46½″	88°,48′, 5½″
S o =	106677, 6	106717, 4
N o =	142879, 7	143138, 1
N o + S o =	249557, 3	249855, 5
N o − S o =	36202, 1	36420, 7
A l(N o − S o) =	4, 5587338	4, 5613482
ſubt. l(N o + S o) =	5, 3971703	5, 3976889
	9, 1615635	9, 1636583
add l tang (90 − $\frac{1}{2}$ S $o\zeta$) =	11, 6775226	11, 6794316
l tang $\frac{1}{2}$ (oSN − SNo) =	10, 8390861	10, 8430899
$\frac{1}{2}$ (oSN − SNo) =	81°,45′,29″½	81°,49′,58″
$\frac{1}{2}$ (oSN + SNo) =	88, 47, 46 ½	88, 48, 5½
oSN =	170, 33, 16	170, 38, 3½
SNo =	7, 2, 17	6, 58, 7½
ab l ſin S$o\zeta$ =	8, 6233157	8, 6214086
ſubtr. l ſin oSN =	9, 2151360	9, 2114815
	9, 4081797	9, 4099271
add. lNo =	5, 1549705	5, 1557552
lSN =	4, 5631502	4, 5656823
Ang. gSη =	37°,24′,14″	37°,26′,53″
add oSN =	5ˢ,20, 33, 16	5ˢ,20, 38, 3
ſubtr.	6ˢ,27, 57, 30	6ˢ,28, 4, 56″
6ˢ + g =	16, 0, 29, 2	16, 0, 29 2

Long. Nodi Aſcend.	9s,2°,31′,32″	9s,2°,24′, 6″
l tang HNθ =	9, 3188523	9, 3186645
ſubtr. *l* ſin SN*o* =	9, 0882372	9, 0839610
l tang. Inclinat.	10, 2306151	10, 2347035
Inclin. Orbitæ ad Eclipt. =	59°,32′,38″	59°,46′,45″
l coſ SN*o* =	9, 9967152	9, 9967798
add. *l* coſ HNθ =	9, 9907700	9, 9907779
l coſ SNH =	9, 9874852	9, 9875577
Ergo SNH =	13°,41′,19″	13°,38′,59″
a { *l* F ς =	4, 4451122	4, 4457037
l H θ =	4, 5005220	4, 5011245
ſubtr. *l* ſin HNθ =	9, 3096223	9, 3094424
l FN =	5, 1354899	5, 1362613
l HN =	5, 1908997	5, 1916821
½ SNH =	6°, 50′,39½″	6°, 49′,29½″
90 − ½ SNH =	83°, 9′,20½″	83°,10,′30½″
SN =	36572, 12	36785, 97
NF =	136612, 3	136855, 2
NF + SN =	173184, 4	173641, 2
NF − SN =	100040, 2	100069, 2
a *l* (NF − SN) =	5, 0001745	5, 0003004
ſubt. (*l* NF + SN) =	5, 2385087	5, 2396528
	9, 7616658	9, 7606476
add *l* tang (90 − ½ SNH) =	10, 9207206	10, 9219679
l tang ½ (NSF − NFS) =	10, 6823864	10, 6826155
½ (NSF − NFS) =	78°,15′,42½″	78°,16′,4⅓″
½ (NSF + NFS) =	83, 9, 20½	83, 10, 30½
NSF =	161, 25, 3	161, 26, 35
NFS =	4, 53, 38	4, 54, 26⅙
A *l* ſin SNH =	9, 3740724	9, 37288[illegible]5
ſubtr *l* ſin NFS =	8, 9310033	8, 9321860

add.

	0, 4430691	0, 4406993
add. lSN =	4, 5631502	4, 5656823
lSF =	5, 0062193	5, 0063816
NH =	155202, 8	155482, 7
SN =	36572, 12	36785, 97
NH+SN =	191774, 9	192268, 7
NH − SN =	118630, 7	118696, 7
A l(NH − SN) =	5, 0741971	5, 0744387
ſubt. l(NH+SN) =	5, 2827918	5, 2839086
	9, 7914053	9, 7905301
add l tang (90 − $\frac{1}{2}$ SNH) =	10, 9207206	10, 9219679
l tang $\frac{1}{2}$ (NSH − NHS) =	10, 7121259	10, 7124980
$\frac{1}{2}$ (NSH − NHS) =	79°,1′,9$\frac{1}{6}$″	79°,1′,42$\frac{1}{2}$
$\frac{1}{2}$ (NSH+NHS) =	83, 9, 20$\frac{1}{2}$	83,10, 30$\frac{1}{2}$
NSH =	162, 10,29$\frac{2}{3}$	162, 12,13
NHS =	4, 8,11$\frac{1}{3}$	4, 8,48
A l ſin SNH =	9, 3740724	9, 3728853
ſubtr. l ſin NHS =	8, 8581311	8, 8591973
	0, 5159413	0, 5136880
Add lSN =	4, 5631502	4, 5656823
lSH =	5, 0790915	5, 0793703
NSH =	162°,10′,29$\frac{2}{3}$″	162°,12′,13″
NSF =	161, 25, 3	161, 26, 35
FSH =	0°,45′,26$\frac{2}{3}$″	0°,45′,38″
FS = y =	101442, 3	101480, 3
HS = z =	119975, 2	120052, 2
lT =	0, 9056124	0, 9056124
φ =	0°,45′,26$\frac{2}{3}$″	0°, 45′,38″
l ſin φ =	8, 1211936	8, 1229950
l coſ φ =	9, 9999635	9, 9999638

l cot φ =

$l \cot \phi =$	11, 8787699	11, 8769688
$l y =$	5, 0062193	5, 0063816
$l z =$	5, 0790915	5, 0793703
$l yz =$	10, 0853108	10, 0857519
$l yyzz =$	20, 1706216	20, 1715038
$l 4 =$	0, 6020600	
$l m^2 =$	10, 8691050	
$l T^2 =$	1, 8112248	
$l 4 m^2 T^2 =$	13, 2823898	13, 2823898
$l \frac{yyzz}{4m^2T^2} =$	6, 8882318	6, 8891140
add. $2 l \sin \phi =$	16, 2423872	16, 2459900
	3, 1306190	3, 1351040
Pars prior =	1350, 887	1364, 910
$l \sqrt{yz} =$	5, 0426554	5, 0428759
ſubt. $l 3 =$	0, 4771213	0, 4771213
	4, 5655341	4, 5657546
add $2 l \phi =$	16, 2423872	16, 2459900
	0, 8079213	0, 8117446
pars poſt. =	6, 426	6, 482
Ergo $b =$	1357, 313	1371, 392
$y =$	101442, 3	101480, 3
$z =$	119975, 2	120052, 2
$y - b =$	100085, 0	100108, 9
$z - b =$	118617, 9	118680, 8
A $l(z-b) =$	5, 0741502	5, 0743804
ſubtr. $l(y-b) =$	5, 0003690	5, 0004728
	0, 0737812	0, 0739076
Add $l \frac{y}{z} =$	9, 9271278	9, 9270113

ſubtr.

	10, 0009090	10, 0009189
ſubt. l ſin ϕ =	8, 1211936	8, 1229950
l poſt. ſubtr.	1, 8797154	1, 8779239
Paſſ ſubtrah. =	75, 80807	75, 49600
a cot ϕ =	75, 64322	75, 33014
— tang v =	0, 16485	0, 16586
Ergo — v =	9°,21′,40″	9°,25′, 4″
Anom. vera loci F ſeu v =	170°,38′,20″	170°,34′,56″
ſubtr. FSN =	161°,25′, 3″	161°,26′,35″
Diſt. Nodi ☊ a perihelio	9°,13′,17″	9°, 8′,21″
Ad. ly =	5, 0062193	5, 0063816
add. l – coſ v =	9, 9941775	9, 9941065
l — y coſ v =	5, 0003968	5, 0004881
add. lb =	3, 1326800	3, 1371615
l Num =	8, 1330768	8, 1376496
– y coſ v =	100091, 41	100112, 45
add. $y - b$ =	100085, 0	100108, 9
Denom.	200176, 4	200221, 3
l Denom. =	5, 3014129	5, 3015103
la = =	2, 8316639	2, ~~8361~~393
Diſtantia perih. a Sole a =	678, 678	685, 708
Ergo $2a$ =	1357, 356	1371, 416
& b =	1357, 313	1371, 392
$\delta = 2a - b$ =	0, 043	0, 024

Eſt ergo orbita cometæ utroque caſu Ellipſis maxime oblonga & parabolæ proxima;

$a\,l\delta$ =	(–2), 6334685	(–2), 3802112
ſubtr. lb =	3, 1326800	3, 1371615
$n = ln$ =	5, 5007885	5, 2430497 Charact. 10. minuatur.

$\frac{1}{2} v =$

$\frac{1}{2} v$	85°,19′,10″	85°,17′,28″
$l\,t$	1, 0868576	1, 0842248
n	0, 0000316	0, 0000175
$l\,t^3$	3, 2605728	3, 2526744
$l\,t^5$	5, 4342880	5, 4211240
$l\,t^7$	7, 6080032	7, 5895736
$l\,n\,t^5$	0, 9350765	0, 6641737
$l\,n\,n\,t^7$	(−2), 6095802	(−2), 0756730
t	12, 21399	12, 14017
$\frac{1}{3} t^3$	607, 36750	596, 4215
$+ \frac{1}{3} t^3$	619, 5815	608, 5616
ſubtr.	3, 4445	1, 8460
add.	0, 0174	0, 0051
$t + \frac{1}{3}t^3 - \frac{3}{5}n t^5 + \frac{3}{7} n^2 t^7 -$ &c.	616, 1544	606, 7207
hujus log.	2, 7896895	2, 7829888
$2\,la$	5, 6633278	5, 6722786
ſubtr. lVb	1, 5663400	1, 5685808
	4, 0969878	4, 1036978
ſubtr. lm	5, 4345525	5, 4345525
	8, 6624353	8, 6691473
add.	2, 7896895	2, 7829888
	1, 4521248	1, 4521361
A Perih. ad F dies.	28, 3221	28, 3228
ſeu	$28^d,7^h,43',50''$	$28^d,7^h,44',50''$
At eſt tempus in F A 1680 Dec.	$36^d,6,$ 1, 38	$39^d,9$ 1, 38
Cometa per perihelium tranſiit A. 1680 Decembr.	$7^d,22^h,17',48''$	$7^d,22^h,16',4$[illegible]
Diſtantia Perih. a Sole	678, 678	685, 708
ſemilatus rectum	1357, 313	1371, 39[illegible]
Diſt. Nodi ☊ a Perihelio	9°,13′,17″	9°, 8′,21″
Longitudo Nodi ☊	9^s,2°,31′,32″	9^s,2°,24′,[illegible]6″
Inclinatio Orbitæ ad Eclipt.	59°,32′,38″	59°,46′,45″

Compa-

Comparentur jam hæ binæ determinationes cum obſervatione quadam quarta; quem in finem eligatur obſervatio A. 1680 die 12 Decembris facta, quæ ita ſe habebat.

A. 1680. Dec. $12^d, 4^h, 46', 0''$

Longitudo Cometæ; $9^s, 6°, 31', 21''$

Latitudo Cometæ bor. $8°, 26', 0''$

Longitudo Solis $9^s, 1°, 51', 23''$

Diſt. Solis a terra = 98275, ejus log. = 4,9924431

Temp. propoſitum A. 1680 Mens. Dec.	$12^d, 4^h, 46', 0''$	$12^d, 4^h, 46', 0''$
ſubtr. temp. Perihelii	7, 22, 17, 48	7, 22, 16, 48''
Erit tempus T =	4, 6, 28, 12	4, 6, 29, 12
& in diebus eſt T =	4, 26958	4, 27027
lT =	0, 6303852	0, 6304553
add $l\,m$ =	5, 4345525	5, 4345525
	6, 0649377	6, 0650078
ſubtr. $l \frac{aa}{\sqrt{b}}$ =	4, 0969878	4, 1036978
Vid. Probl. VII. $l\,n$ =	1, 9679499	1, 9613100
& n =	92, 88592	91, 47660
$\frac{1}{2} n$ =	46, 44296	45, 73830
Ergo $\frac{3}{2} n$ =	139, 32888	137, 21490
$l \frac{3}{2} n$ =	2, 1440412	2, 1374012
$2 l \frac{3}{2} n$ =	4, 2880824	4, 2748024
$\frac{9}{4} nn$ =	19413, 55	18827, 93
ad 1	19414, 55	18828, 93
$l (\frac{9}{4} nn + 1)$ =	4, 2881271	4, 2748255
$l \sqrt{(\frac{9}{4} nn + 1)}$ =	2, 1440636	2, 1374128
$\sqrt{(\frac{9}{4} nn + 1)}$ =	139, 3361	137, 2185
add. $\frac{3}{2} n$ =	139, 3289	137, 2149

	278, 6650	274, 43[illegible]
$l\,(\tfrac{3}{2}n + \sqrt{(\tfrac{9}{4}nn + 1)}) =$	2, 4450829	2, 4384369
l partis majoris =	0, 8150274	0, 8128123
l partis minoris =	9, 1849725	9, 1871876
Pars major =	6, 531718	6, 498489
Pars minor =	0, 153099	0, 153882
Eritque $\theta =$	6, 378619	6, 344607
$l\,\theta =$	0, 8047267	0, 8024047
$l\,\frac{2\delta}{b} =$	(−5), 8018185	(−5), 5440797
$l\,\frac{2\delta}{b}\,\theta =$	(−4), 6065452	(−4), 3464844
$2\,l\,\theta =$	1, 6094534	1, 6048094
$l\,\frac{2\delta}{b}\,\theta^3 =$	(−2), 2159986	(−3), 9512938
	0, 4771213	0, 4771213
$l\,\frac{2\delta}{3b}\,\theta^3 =$	(−3), 7388773	(−3), 4741725
A tang $\theta =$	81°,5′,24″	81°,2′,35″
ſeu	291924″	291755″
huj. log. =	5, 4652698	5, 4650183
add.	4, 6855749	4, 6855749
	0, 1508447	0, 1505932
$l\,\frac{2\delta}{b} =$	(−5), 8018185	(−5), 5440797
$l\,\frac{2\delta}{b}$ A tang $\theta =$	(−5), 9526632	(−5), 6946729
$\theta =$	6, 378619	6, 344607
ſubtr. $\frac{2\delta}{b}\,\theta =$	0, 000404	0, 000222

add. 2 δ

	6, 378215	6, 344385
add. $\frac{2\delta}{3b}\theta^3$ =	0, 005481	0, 002980
- $\frac{2\delta}{b}$ A tang θ =	0, 000089	0, 000049
= tang $\frac{1}{2}v$ =	6, 383785	6, 347414
l tang $\frac{1}{2}v$ =	10, 8050782	10, 8025969
$\frac{1}{2}v$ =	81°, 5′,50″	81°,2′,49″
& v =	162, 11, 40″	162, 5, 38
b =	1357, 313	1371, 392
a =	678, 678	685, 708
$b - a$ =	678, 635	685, 684
$l\,(b-a)$ =	2, 8316362	2, 8361240
$l\,a$ =	2, 8316639	2, 8361393
$l\,\frac{b-a}{a}$ =	9, 9999723	9, 9999847
l - coſ v =	9, 9786825	9, 9784370
$l\,\frac{b-a}{a}$ =	9, 9999723	9, 9999847
	9, 9786548	9, 9784217
- $\frac{b-a}{a}$ coſ v =	0, 9520392	0, 9515284
Denom. =	0, 0479607	0, 0484715
$l\,b$ = l num. =	3, 1326800	3, 1371615
l Den. =	8, 6808855	8, 6854865
$l\,y$ =	4, 4517945	4, 4516750
Ab v =	162°,11′,40″	162°,5′,38″
ſubtr. diſt. Nodi a Perih. =	9, 13, 17	9, 8, 21
ſ. Diſt. Cometæ a Nodo =	152, 58, 23	152, 57, 17

In triangulo ergo ſphærico ad B rectangulo C N P repræſentet N nodum aſcend. N P eclipticam & N C orbitam cometæ, dantur. Fig. 6.

 N C =

	N C =	152°,58',23"	152°,57', [illegible]
	ang.CNP =	59, 32, 38	59, 46, 4[illegible]
ſin CP = ſin NC.ſin N.	*l* ſin NC =	9, 6574473	9, 6577197
tang NP = coſ N.tang NC.	*l* ſin N =	9, 9355161	9, 9365599
	l ſin C P =	9, 5929634	9, 5942796
	Latitudo heliocentr C P =	23°,3',39"	23°,8',6"
	l coſ N =	9, 7049036	9, 7018564
	l — tang N C =	9, 7076705	9, 7080138
	l — tang N P =	9, 4125741	9, 4098702
	Ergo N P =	165°,30',10"	165°,35',20"
	ſeu N P =	5ˢ,15°,30',10"	5ˢ,15°,35',20"
	Add. longitudo Nodi =	9, 2, 31, 32	9, 2, 24, 6
	Longitudo heliocentrica	2, 18, 1, 42	2, 17, 59, 26
	Longitudo terræ =	3, 1, 51, 23	3, 1, 51, 23
Fig. 7.	Ergo ang. *c* S T =	13, 49, 41	13°, 51, 57
	C S *c* =	23, 3, 39	23, 8, 6
	l *y* = *l* C S =	4, 4517945	4, 4516750
	add. { *l* ſin C S *c* =	9, 5929634	9, 5942796
	{ *l* coſ C S *c* =	9, 9638300	9, 9635903
	l C *c* =	4, 0447579	4, 0459546
	l S *c* =	4, 4156245	4, 4152653
	add. { *l* ſin *c* S T =	9, 3784143	9, 3795762
	{ *l* coſ *c* S T =	9, 9872269	9, 9871565
	l *c* P =	3, 7940388	3, 7948415
	l S P =	4, 4028514	4, 4024218
	S P =	25284, 32	25259, 32
	ab S T =	98275	98275
	Erit T P =	72990, 68	73015, 68
	A *l* *c* P =	3, 7940388	3, 7948415
	ſubtr. *l* T P =	4, 8632675	4, 8634162

l tang.

l tang. S T *c* =	8, 9307713	8, 9314253
S T *c* =	4°,52′,24″	4°,52′,51″
Addatur long. Solis	9ˢ,1,51,23	9ˢ,1,51,23
Long. Com. Geocentrica	9ˢ, 6°,43′,47″	9ˢ, 6°,44′,14″
A *l* T P =	4, 8632675	4, 8634162
ſubtr. *l* coſ S T *c* =	9, 9984272	9, 9984223
l c T =	4, 8648403	4, 8649939
a l C *c* =	4, 0447579	4, 0459546
l tang C T *c* =	9, 1799176	9, 1809607
Latitudo Geocentrica	8°,36′,18″	8°,37′,32″

Accedit ergo prior hypotheſis qua ſumſimus $r = 72700$ propius ad veritatem, ex quo concludimus valorem verum ipſius r multo minorem eſſe debere aſſumto. Atque ex latitudinibus concluditur iſte valor ipſius r diminui debere 835, ob longitudines autem diminui deberet 2763. Medium ergo ſumendo foret valor ipſius $r = 72700 - 1800 = 70900$ Hac itaque poſitione orbita cometæ multo magis a parabola ad figuram ellipticam reducetur, qua cognita hujus cometæ tempus periodicum definiri poterit. Quæ inveſtigatio cum operæ pretium ſit, tribuamus iterum ipſi r duos valores intra quos verus contineatur.

Sit ergo r =	70000	72000
erit $l r$ =	4, 8450980	4, 8573325
add. l ſin η =	9, 6128990	9, 6128990
l coſ η =	9, 9600122	9, 9600122
l G η =	4, 4579970	4, 4702315
$l g \eta$ =	4, 8051102	4, 8173447

$g\,\eta =$

$g\eta =$	63842, 5	65666, [illegible]
$gm =$	13539, 0	13539, 0
$gn =$	5875, 0	5875, 0
$m\eta =$	50303, 5	52127, 6
$n\eta =$	57967, 5	59791, 6
$Sg =$	98407, 0	98407, 0
$g\eta =$	63842, 5	65666, 6
$Sg + g\eta =$	162249, 5	164073, 6
$Sg - g\eta =$	34564, 5	32740, 4
$l\,(Sg - g\eta) =$	4, 5386302	4, 5150840
$l\,(Sg + g\eta) =$	5, 2101833	5, 2150387
	9, 3284469	9, 3000453
l tang $(90 - \frac{1}{2} Sg\eta) =$	10, 0897890	10, 0897890
l tang $\frac{1}{2}\,(g\eta S - gS\eta) =$	9, 4182359	9, 3898343
$\frac{1}{2}\,(g\eta S - gS\eta) =$	14°,40′,46″	13°,47′,12″
$\frac{1}{2}\,(g\eta S + gS\eta) =$	50, 52, 52	50, 52, 52
$g\eta S =$	65, 33, 38	64, 40, 4
$gS\eta =$	36, 12, 6	37, 5, 40
A $l\,Sg$ ſin $Sg\eta =$	14, 9838096	14, 9838096
ſubtr. l ſin $g\eta S =$	9, 9592327	9, 9560926
$l\,S\eta =$	5, 0[illegible]45769	5, 0277170
$l\,G\eta =$	4, 4579970	4, 4702315
l tang $GS\eta =$	9, 4334201	9, 4425145
$GS\eta =$	15°,10′,41″	15°,29′,2″
A $l\,G\eta =$	4, 4579970	4, 4702315
ſubtr. l ſin $GS\eta =$	9, 4180021	9, 4264582
$l\,SG =$	5, 0399949	5, 0437733
	12, 3790825	12, 3790825
ſubtr. 2 $l\,SG =$	10, 0799898	10, 0875466
$l\,GO =$	2, 2990927	2, 2915359
add l coſ $GS\eta =$	9, 9845798	9, 9839443

$l\,\eta\,o =$

$l\,\eta\,o$ =	2, 2836725	2, 2754802
Ergo ηo =	192, 164	188, 573
ab $S\eta$ =	105822, 22	106590, 12
$S\,o$ =	105630, 06	106401, 55
Ang. $g\eta S$ =	65°,33',38''	64°,40',4''
$\zeta\,m\,\eta$ =	9, 54, 8	9, 54, 8
$\zeta\,l\,o$ =	75, 27, 46	74, 34, 12
A $l\,m\,\eta$ =	4, 7015982	4, 7170677
ſubtr. l ſin ζlo =	9, 9858686	9, 9840573
	4, 7157296	4, 7330104
add { l ſin $\zeta m\eta$ =	9, 2354458	9, 2354458
add { l ſin $g\eta S$ =	9, 9592327	9, 9560926
$l\,l\,\eta$ =	3, 9511754	3, 9684562
$l\,m\,l$ =	4, 6749623	4, 6891030
Ergo $l\eta$ =	8936, 67	9299, 43
ſubtr. $\eta\,o$ =	192, 16	188, 57
$l\,o$ =	8744, 51	9110, 86
$l\,l\,o$ =	3, 9417355	3, 9595593
add. $l\,\frac{\alpha+\beta}{\alpha}$ =	0, 2991128	0, 2991128
$l\,\lambda\,\theta$ =	4, 2408483	4, 2586721
Ab ang. ζlo =	75°,27',46''	74°,34',12''
ſubtr. $\zeta k\theta$ =	17, 10, 24	17, 10, 24
$k\theta\lambda$ =	58, 17, 22	57, 23, 48
A $l\lambda\theta$ =	4, 2408483	4, 2586721
ſubtr. l ſin $\zeta k\theta$ =	9, 4702096	9, 4702096
	4, 7706387	4, 7884625
add. { l ſin $\zeta l o$ =	9, 9858686	9, 9840573
add. { l ſin $k\theta\lambda$ =	9, 9297837	9, 9255293

$lk\theta$ =	4, 7565073	4, 77251[illegible]
$lk\lambda$ =	4, 7004224	4, 7139918
ml =	47311, 02	48876, 81
add. km =	3285, 17	3285, 17
kl =	50596, 19	52161, 98
ſubtr. $k\lambda$ =	50167, 50	51759, 70
$l\lambda$ =	428, 69	402, 28
$ll\lambda$ =	2, 6321433	2, 6045284
add. $l\,\frac{a}{\beta}$ =	0, 0038429	0, 0038429
$ll\zeta$ =	2, 6359862	2, 6083713
kl =	50596, 19	52161, 98
$l\zeta$ =	432, 50	405, 85
$k\zeta$ =	51028, 69	52567, 83
$k\theta$ =	57082, 07	59227, 02
$k\theta + k\zeta$ =	108110, 76	111794, 85
$k\theta - k\zeta$ =	6053, 38	6659, 19
$l\,(k\theta - k\zeta)$ =	3, 7819979	3, 8234215
$l\,(k\theta + k\zeta)$ =	5, 0338690	5, 0484219
	8, 7481289	8, 7749996
l tang $(90 - \frac{1}{2}\zeta k\theta)$ =	10, 8210294	10, 8210294
	9, 5691583	9, 5960290
$\frac{1}{2}(k\zeta\theta - k\theta\zeta)$ =	20°,20′,44″	21°,31′,42″
$\frac{1}{2}(k\zeta\theta + k\theta\zeta)$ =	81, 24, 48	81, 24, 48
$k\zeta\theta$ =	101, 45, 32	102, 56, 30
$k\theta\zeta$ =	61, 4, 4	59, 53, 6
ζlo =	75°,27, 46	74, 34, 12
$k\zeta\theta + \zeta lo$ =	177, 13, 18	177, 30, 42
$So\zeta$ =	2, 46, 42	2, 29, 18
A l ſin $\zeta k\theta$ =	9, 4702096	9, 4702096
ſubtr. l ſin $k\zeta\theta$ =	9, 9907889	9, 9888258

add. $lk\theta$

	9, 4794207	9, 4813838
add. $lk\theta$ =	4, 7565073	4, 7725198
$l\vartheta\zeta$ =	4, 2359280	4, 2539036
fm =	6672, 1	6672, 1
ml =	47311, 0	48876, 8
$l\zeta$ =	432, 5	405, 8
Erit $f\zeta$ =	54415, 6	55954, 8
hk =	17245, 1	17245, 1
$k\theta$ =	57082, 1	59227, 0
erit $h\theta$ =	74327, 2	76472, 1
$lf\zeta$ =	4, 7357234	4, 7478373
l tang ζ =	9, 6931129	9, 6931129
$lF\zeta$ =	4, 4288363	4, 4409502
$lh\theta$ =	4, 8711478	4, 8835030
l tang θ =	9, 6127775	9, 6127775
$lH\theta$ =	4, 4839253	4, 4962805
Ergo $H\theta$ =	30473, 7	31353, 1
$F\zeta$ =	26843, 3	27602, 6
$H\theta - F\zeta$ =	3630, 4	3750, 5
$l(H\theta - F\zeta)$ =	3, 5599545	3, 5740892
[illegible] θ =	4, 2359280	4, 2539036
l tang $HN\theta$ =	9, 3240265	9, 3201856
a $lH\theta$ =	4, 4839253	4, 4962805
$lN\theta$ =	5, 1598988	5, 1760949
Ang. $HN\theta$ =	11°,54′,28″	11°,48′,21″
$N\theta$ =	144510, 3	150001, 3
$Al\zeta\vartheta$ =	4, 2359280	4, 2539036
ſubtr. $l\,\frac{\alpha+\beta}{\beta}$ =	0, 3029557	0, 3029557
$l\vartheta o$ =	3, 9329723	3, 9509479
θo =	8569, 83	8931, 98

N*o* =	135940, 5	141069, 3
½ S*o*ζ =	1°,23′,21″	1°,14′,39″
90 — ½ S*o*ζ =	88, 36, 39	88, 45, 21
S*o* =	105630, 06	106401, 55
N*o* =	135940, 5	141069, 3
N*o* + S*o* =	241570, 5	247470, 8
N*o* — S*o* =	30310, 5	34667, 8
l (N*o* — S*o*) =	4, 4815930	4, 5399263
l (N*o* + S*o*) =	5, 3830439	5, 3935239
	9, 0985491	9, 1464024
l tang (90 — ½ S*o*ζ) =	11, 6152831	11, 6631758
l tang ½ (*o*SN — SN*o*) =	10, 7138322	10, 8095782
½ (*o*SN — SN*o*) =	79°, 3′,40″	81°,11′,15″
½ (*o*SN + SN*o*) =	88, 36, 39	88, 45, 21
*o*SN =	167, 40, 19	169, 56, 36
SN*o* =	9, 32, 59	7, 34, 6
ab *l* ſin S*o*ζ =	8, 6854914	8, 6376495
ſubtr. *l* ſin *o*SN =	9, 3294159	9, 2420989
	9, 3560755	9, 3955506
l N*o* =	5, 1333489	5, 1494324
l SN =	4, 4894244	4, 5449830
Ang. gSη =	36, 12, 6	37, 5, 40
ſeu	1ˢ,6°, 12′,6″	1ˢ,7°, 5′,40″
*o*SN =	5, 17,40,19	5, 19,56,36
	6, 23,52,25	6, 27, 2,16
a 6ˢ + *g* =	16, 0, 29, 2	16, 0, 29, 2
Long. Nodi aſcend.	9, 6, 36,37	9, 3, 26,46
l tang HNϑ =	9, 3240265	9, 3201856
ſubtr. *l* ſin SN*o* =	9, 2198555	9, 1196138
	10, 1041710	10, 2005718

Inclinatio

Inclinatio Orb. ad Eclipt.	51°,48′,24″	57°,47′,2″
l coſ SN*o* =	9, 9939394	9, 9962000
add. *l* coſ HNϑ =	9, 9905525	9, 9907146
l coſ SNH =	9, 9844919	9, 9869146
SNH =	15°,13′,15″	13°,59′,40″
a { *l*F♋ =	4, 4288363	4, 4409502
*l*Hϑ =	4, 4839253	4, 4962805
ſubtr. *l* ſin HNϑ =	9, 3145790	9, 3109002
*l*FN =	5, 1142573	5, 1300500
*l*HN =	5, 1693463	5, 1853803
½ SNH =	7°,36′,37½″	6°,59′,50″
90 — ½ SNH =	82, 23, 22½	83, 0, 10″
SN =	30862, 02	35073, 82
NF =	130094, 03	134911, 87
NF + SN =	160956, 05	169985, 7
NF − SN =	99232, 01	99838, 05
l (NF − SN) =	4, 9966517	4, 9992961
l (NF + SN) =	5, 2067073	5, 2304124
	9, 7899444	9, 7688837
l tang (90 — ½ SNH) =	10, 8741495	10, 9110303
l tang ½ (NSF − NFS) =	10, 6640939	10, 6799140
½ (NSF − NFS) =	77°,46′,18½″	78°,11′,48″
½ (NSF + NFS) =	82, 23, 22½	83, 0, 10
NSF =	160, 9, 41	161, 11, 58
NFS =	4, 37, 4	4, 48, 22
l ſin SNH =	9, 4191955	9, 3835062
ſubt. *l* ſin NFS =	8, 9058402	8, 9231624
	0, 5133553	0, 4603438
*l*SN =	4, 4894244	4, 5449830
*l*SF =	5, 0027797	5, 0053268

NH =	147688, 39	153242, 89
SN =	30862, 02	35073, 82
NH+SN =	178550, 41	188316, 71
NH − SN =	116826, 37	118169, 07
a *l* (NH − SN) =	5, 0675410	5, 0725038
l (NH+SN) =	5, 2517608	5, 2748889
	9, 8157802	9, 7976149
l tang (90−½SNH) =	10, 8741495	10, 9110303
	10, 6899297	10, 7086452
½ (NSH − NHS) =	78°,27′,30⅔″	78°,55′,59¼
½ (NSH+NHS) =	82, 23, 22½	83, 0, 10
NSH =	160, 50, 53⅙	161, 56, 9 ¼
NSF =	160, 9, 41	161, 11, 58
FSH =	0, 41, 12⅙	0, 44, 11¼
NHS =	3, 55, 52	4, 4, 11
A *l* ſin SNH =	9, 4191955	9, 3835062
ſubtr. *l* ſin NHS =	8, 8360518	8, 8510771
	0, 5831437	0, 5324291
*l*SN =	4, 4894244	4, 5449830
*l*SH =	5, 0725681	5, 0774121
FS = *y* =	100642, 1	101234, 1
HS = *z* =	118186, 6	119512, 2
Φ =	0°, 41′,12⅙	0, 44′,11¼″
l ſin Φ =	8, 0786123	8, 1090132
l coſ Φ =	9, 9999688	9, 9999641
l cot Φ =	11, 9213565	11, 8909509
ly =	5, 0027797	5, 0053268
lz =	5, 0725681	5, 0774121
lyz =	10, 0753478	10, 0827389

lηηzz =

$lyyzz$ =	20, 1506956	20, 1654778
ſubtr.	13, 2823898	13, 2823898
	6, 8683058	6, 8830880
add $2\,l$ ſin Φ =	16, 1572246	16, 2180264
	3, 0255304	3, 1011144
Pars prior.	1060, 548	1262, 160
$l\sqrt{yz}$ =	5, 0376739	5, 0413694
	0, 4771213	0, 4771213
	4, 5605526	4, 5642481
$2\,l$ ſin Φ =	16, 1572246	16, 2180264
	0, 7177772	0, 7822745
Pars poſter. =	5, 221	6, 057
Ergo b =	1065, 769	1268, 217
y =	100642, 1	101234, 1
z =	118186, 6	119512, 2
$y-b$ =	99576, 4	99965, 9
$z-b$ =	117120, 9	118244, 0
A $l\,(z-b)$ =	5, 0686344	5, 0727790
ſubtr. $l\,(y-b)$ =	4, 9981564	4, 9998519
	0, 0704780	0, 0729271
add $l\,\frac{y}{z}$ =	9, 9302116	9, 9279147
	10, 0006896	10, 0008418
ſubtr. l ſin Φ =	8, 0786123	8, 1090132
	1, 9220773	1, 8918286
Pars ſubtr. =	83, 57518	77, 95224
a cot. Φ =	83, 43658	77, 79487
— tang v =	0, 13860	0, 15737
Anom. vera loci F = v =	172°, 6′, 32″	171°, 3′, 24″
ſubtr. FSN =	160, 9, 41	161°, 11, 58

Diſt. Nodi

Diſt. Nodi ☊ a Perihel.	11, 56, 51	9, 51, 26
Ad ly =	5, 0027797	5, 0053268
add l − coſ v =	9, 9958679	9, 9946878
l — y coſ v =	4, 9986476	5, 0000146
add lb =	3, 0276634	3, 1031935
l Num.	8, 0263110	8, 1032081
− y coſ v =	99689, 10	100003, 36
y − b =	99576, 4	99965, 9
Den.	199265, 5	199969, 2
l Denom.	5, 2994320	5, 3009631
la =	2, 7268790	2, 8022450
Diſt. Perih. a Sole = a =	533, 1864	634, 2274
Ergo $2a$ =	1066, 3728	1268, 4548
b =	1065, 769	1268, 217
δ = $2a-b$ =	0, 603	0, 237
$l\delta$ =	9, 7803173	9, 3747483
lb =	3, 0276634	3, 1031935
Char. 10 minuenda. ln =	6, 7526539	6, 2715548
$\frac{1}{2}v$ =	86°, 3′, 16″	85°, 31′, 42″
lt =	1, 1613272	1, 1067702
lt^3 =	3, 4839816	3, 3203106
lt^5 =	5, 8066360	5, 5338510
lt^7 =	8, 1292904	7, 7473914
lnn =	13, 5053078	12, 5431096
lnt^5 =	2, 5592899	1, 8054058
ln^2t^5 =	9, 3119438	8, 0769606
ln^2t^7 =	1, 6345982	0, 2905010
ln^3t^7 =	8, 3872521	6, 5620558
ln^3t^9 =	9, 5485793	7, 6688260

t =

$t =$	14, 4986	12, 7870
$\frac{1}{3}t^3 =$	1015, 9220	696, 9303
$\frac{3}{5}n^2t^5 =$	0, 1231	71
$\frac{3}{7}n^2t^7 =$	18, 4766	8366
	1049, 0203	710, 5610
ſubtr. $\frac{2}{5}nt^5 =$	144, 9940	25, 5544
	904, 0263	685, 0066
ſubtr. $\frac{4}{7}n^3t^7 =$	139	2
$\frac{4}{9}n^3t^9 =$	1572	20
$t + \frac{1}{3}t^3 +$ &c.	903, 8552	685, 0044
$2 la =$	5, 4537580	5, 6044900
$l\sqrt{b} =$	1, 5138317	1, 5515967
	3, 9399263	4, 0528933
$lm =$	5, 4345525	5, 4345525
	8, 5053738	8, 6183408
add. $l(t + \frac{1}{3}t^3 +$ &c.$) =$	2, 9560989	2, 8356934
	1, 4614727	1, 4540342
A Perih. ad F dies	28, 9383	28, 4468
ſeu conſueto more	$28^d, 22^h, 31', 9''$	$28^d, 10^h, 43', 23''$
Com. in F. A. 1680. M. Dec.	36, 6, 1', 38	36, 6, 1, 38
Cometa per Perihelium tranſiit A. 1680. M. Dec. =	$7^d, 7^h, 30', 29''$	$7^d, 19^h, 18', 15''$
Diſt. Perihelii a Sole a =	533, 1864	634, 2274
ſemilatus rectum b =	1065, 769	1268, 217
Diſt. Nodi ☊ a Perihelio =	$11^\circ, 56', 51''$	$9^\circ, 51', 26''$
Long. Nodi ☊ helioc.	$9^s, 6^\circ, 36', 37''$	$9^s, 3^\circ, 26', 46''$
Inclinatio ad Eclipticam	$51^\circ, 48', 24''$	$57^\circ, 47', 2''$

Computetur nunc pari modo locus cometæ geocentricus.

ad A. 1680. M. Dec.	$12^d, 4^h, 46', 0''$	$12^d, 4^h, 46', 0''$
Tempus Perih. ſubtr.	7, 7, 30, 29	7, 18, 19, 15

T =

$T =$	4, 21,15,30	4, 9,27,45
in diebus autem eſt $T =$	4, 8857	4, 3943
Ad Probl. VIII. $l\,(2a-b) =$	9, 7803173	9, 3747482
ſemiſs.	9, 8901586	9, 6873742
$\frac{3}{2}\,l\,(2a-b) =$	9, 6704759	9, 0621225
$l\,T =$	0, 6889268	0, 6428897
add.	11, 0500076	11, 0500076
	11, 4094103	10, 7550198
ſubtr. $3\,l\,a =$	8, 1806370	8, 4067350
$l\,u =$	3, 2287733	2, 3482848
Ergo $u =$	1693,″4	223,″0
ſeu Anomalia media $u =$	0°,28′,13″	0°,3′,43″
$b-a =$	532, 583	633, 990
$l\,(b-a) =$	2, 7263872	2, 8020824
$l\,a =$	2, 7268790	2, 8022450
$l\,\frac{b-a}{a} =$	9, 9995082	9, 9998374
ſit $\varrho =$	21°, 30′	10°, 30′
l ſin $\varrho =$	9, 5640754	9, 2606330
add.	5, 3139333	5, 3142625
$l\,\frac{(b-a)}{a}$ ſin $\varrho =$	4, 8780087	4, 5748955
$\frac{b-a}{a}$ ſin $\varrho =$	75510,″7	37574,″7
add. $u =$	1693, 4	223, 0
	77204, 1	37797, 7
$u + \frac{b-a}{a}$ ſin $\varrho =$	21°, 26′,44″	10°, 29′,57″
ſubtr. $\varrho =$	21°, 30′, 0″	10°, 30′, 0″

num. = –

num. = —	$-3', 16''$	$-3''$
l coſ ϱ =	9, 9686779	9, 9926661
$l\,\frac{(b-a)}{a}$ =	9, 9995082	9, 9998374
	9, 9681861	9, 9925035
$1-\frac{(b-a)}{a}$ =	0, 070634	0, 017112
erit z =	$-46',15''$	$-2', 55''$
Erit ω =	$20°, 43',5''$	$10°,27', 5''$
$\frac{1}{2}\,\omega$ =	$10°, 21',32\frac{1}{2}$	$5°, 13',32\frac{1}{2}''$
l tang $\frac{1}{2}\,\omega$ =	9, 2619649	8, 9612284
ſubtr. $l\sqrt{\frac{2a-b}{b}}$ =	8, 3763269	8, 1357774
l tang $\frac{1}{2}\,v$ =	10, 8856380	10, 8254510
Ergo $\frac{1}{2}\,v$ =	$82°, 35',9\frac{1}{2}''$	$81°,29',56\frac{1}{2}''$
Et anomalia vera v =	165, 10, 19	$162,59',53$
l-coſ v =	9, 9852909	9, 9805920
$l\,\frac{b-a}{a}$ =	9, 9995082	9, 9998374
	9, 9847991	9, 9804294
$-\frac{(b-a)}{a}$ coſ v =	0, 965604	0, 955937
Den. =	0, 034395	0, 044062
$l\,b$ =	3, 0276634	3, 1031935
lDen. =	8, 5364953	8, 6440642
$l\,y$ =	4, 4911681	4, 4591293
Ab v =	$165°,10',10''$	$162°,59',53''$
Diſt. ☊ a Perih.	11, 56 ,51	9, 51, 26
N C = Diſt. Com. a ☊ =	$153°, 13',28''$	$153°, 8',27''$
CNP =	$51°, 48',24''$	57, 47, 2

Fig. 6.

lſin N C =

l ſin NC =	9, 6536916	9, 6549454
l ſin N =	9, 8953998	5, 9273925
l ſin CP =	9, 5490914	9, 5823379
l coſ N =	9, 7911844	9, 7268202
l—tang NC =	9, 7029480	9, 7045221
l—tang NP =	9, 4941324	9, 4313423
Lat. helioc. CP =	20°,44′,12″	22°,28′,21″
Ergo NP =	162, 40, 22	164, 53, 28
ſeu NP =	5ˢ,12°, 40,22	5ˢ, 14,53,28
Add. long. ☊ =	9, 6, 36,37	9, 3,26,46
Longitudo heliocentr.	2, 19,16,59	2ˢ,18,20,14
Longitudo terræ	3, 1,51,23	3, 1,51,23
Ang. *c*ST =	12°,34,24″	13°,31′, 6″
CS*c* =	20, 44, 12	22, 28, 21
ly=*l*CS =	4, 4911681	4, 4591293
add. *l* ſin CS*c* =	9, 5490914	9, 5823379
add. *l* coſ CS*c* =	9, 9709127	9, 9657017
l C*c* =	4, 0402595	4, 0414672
l S*c* =	4, 4620808	4, 4248310
add. { *l* ſin *c*ST =	9, 3378364	9, 3687698
add. { *l* coſ *c*ST =	9, 9894579	9, 9877967
*lc*P =	3, 7999172	4, 7936208
*l*SP =	4, 4515387	4, 4126277
SP =	28283, 9	25859, 9
ab ST =	98275,	98275, 0
TP =	69991	72415,
*a lc*P =	3, 7999172	3, 7936208
ſubtr. *l* TP =	4, 8450422	4, 8598285
l tang ST*c* =	8, 9548750	8, 9337923
ST*c* =	5°,9′,1″	4°,54′,27″
Long. Solis	9ˢ,1, 51, 23	9, 1, 51, 23

Long.

Long. Com. Geocentr.	9, 7, 0, 24	9, 6, 45, 50
a lTP =	4, 8450422	4, 8598285
ſubtr. l. coſ STc =	9, 9982431	9, 9984053
l cT =	4, 8467991	4, 8614232
a lCc =	4, 0402595	4, 0414672
l tang CTc =	9, 1934604	9, 1800440
Latitudo Geocentr. =	8°, 52′, 24″	8°, 36′, 27″

Hinc tam ex longitudinibus quam latitudinibus obſervatis liquet verum valorem ipſius r majorem eſſe quam 72000. Ex præcedente autem hypotheſi collegimus, r minorem eſſe quam 72700. Sunt autem omnes quatuor tam longitudines quam latitudines obſervatis majores, ex quo ſequitur, valores intra 72000 & 72700 contentos præbituros eſſe minores & longitudines & latitudines, ſicque veritati magis conſentaneas; ita ut intra hos limites minima longitudo & latitudo contineatur. Cum autem valores 72000 & 72700 pro r ſubſtituti æquales fere præbeant longitudines ac latitudines, minimum fere reſpondebit valori medio 72350; quem tanquam verum ipſius r valorem aſſumamus; cum eum propius definire vix liceat.

Comparemus ergo has duas hypotheſes inter ſe.

Hypotheſis r =	72000	72700
Cometa per perihelium tranſiit A. 1680 M Dec.	7^d; 19^h, 18′, 15″	7^d, 22^h, 17′, 48″
Diſt. Perih. a ſole – –	634, 227	678, 678
Semi latus rectum – –	1268, 217	1357, 313
Diſt. nodi ☊ a Perih. –	9°, 51′, 26	9°, 13′, 17″
Long. nodi ☊ helioc. –	9^s, 3°, 26, 46	9^s, 2°, 31′, 32″
Inclin. Orbitæ ad Eclipt.	57°, 47′, 2″	59°, 32′, 38″

Sumendo ergo inter hæc ſyſtemata medium, prodibit
Cometæ hujus ſequens theoria quaſi vera

I. Cometa per perihelium tranſiit A. 1680 M. Decembri	–	$7^d, 20^h, 48', 0''$
II. Diſtantia Perihelii a Sole	=	656, 4525
III. Semi-latus rectum	=	1312, 7650
IV. Diſtantia nodi ☊ a Perih.	=	9°, 32', 21''
V. Longitudo helioc. nodi ☊	=	9^s, 2°, 59', 9'
VI. Inclinatio Orbitæ ad Eclipt.	=	58°, 39', 50''

Non multum discrepant hæ orbitæ Cometæ determinationes ab iis, quas Neutonus & Hallejus invenerunt, quarum ille conſtructione geometrica, hic vero calculo eſt uſus, ita tamen ut locum nodorum, & inclinationem & tempus, quo per perihelium tranſiit, eadem retinuerit, quæ Neutonus per conſtructionem inveniſſet: uterque autem orbitam cometæ perfectam eſſe parabolam aſſumſit. Quare cum conſtructio geometrica in hoc negotio admodum ſit lubrica, atque Hallejus præcipua capita hinc nata retinuerit, mirum non eſt, quendam diſſenſum inter hanc orbitæ determinationem & Neutonianam deprehendi, neque enim eſſet mirandum, ſi iſte disſenſus major prodiiſſet. Statuit autem Neutonus I. hunc Cometam in perihelio eſſe verſatum A. 1680. M. Dec. $8^d, 0^h, 4'$; II. diſtantiam perihelii a ſole = 607, 5; III. ſemi-latus rectum = 1215. IV. Diſtantiam nodi a perihelio = 9°, 20': V. Long. nodi ☊ = 9^s, 1°, 53' & VI. inclinationem orbitæ ad Eclipt. 61°, 20', 20''. Orbita autem hujus cometæ ſic proxime inventa poterit ſecundum

dum methodum, quam in ſuperiore diſſertatione de Cometa A. 1742. tradidi, ulterius corrigi, atque cum accuratiſſimæ obſervationes per ſatis longum temporis ſpatium ſuppetant, fere ad [illegible] perfectionis gradum perduci, ut axis transverſus ejus orbitæ definiri, atque adeo ejus tempus revolutionis asſignari queat. Quem laborem aliis relinquens interim ex hac cometæ orbita inventa, tanquam eſſet vera, tempus periodicum inveſtigabo. Cum igitur ſit

Diſtantia perihelii a ſole a	=	656, 4525
erit $2a$	=	1312, 9050
ſubtrahatur b	=	1312, 7650
erit $2a - b$	=	0, 1400
At eſt la	=	2, 8172032
unde laa	=	5, 6344064
ſubtr. $l(2a-b)$	=	9, 1461280
l ſemi- axis transverſi	=	6, 4882784
addatur ſemiſſis	=	3, 2441392
		9, 7324176
ſubtrahatur $lc\sqrt{c}$	=	7, 5000000
		2, 2324176

Unde erit tempus periodicum = 170, 77 annorum. Reverteretur itaque iſte cometa ad perihelium poſt annos centenos & ſeptuaginta; & quamvis exiguum diſcrimen in valore $2a-b$ vehementer hoc tempus immutare poſſit, tamen non adeo a veritate aberrabit. Neque enim contra hanc reverſionem quicquam valet objectio, quod idem cometa annis 170 antea non ſit obſervatus, neque in Tabula Halleji unquam ante ullus ſit obſervatus, qui cum hoc conſentiret. Primum enim

ſtatus

ſtatus cometæ apparens plurimum a loco terræ in orbita ſua pendet, quæ ſi verſetur in iisdem ſignis, quibus cometa eſt proximus, cometa maxime fulgens conſpicietur, & caudæ longitudo apparens non ex ejus vera quantitate, ſed potiſſimum e ſitu relativo reſpectu noſtri æſtimatur. Utraque opportunitas locum habuit A. 1680, quo tempore non ſolum cometa ſatis
Fig. 8. prope ad locum terræ acceſſit, ſed etiam caudæ directio C *c* ita ratione terræ T erat diſpoſita, ut ejus magnitudo apparens ſeu angulus C T *c*, fuerit vehementer magnus; quanquam & hic cometa ob ſummam in perihelio vicinitatem cum ſole præ reliquis multo majori cauda inſtructus eſt putandus. Ex his autem intelligitur, ſi idem hic cometa vel in poſterum redeat, vel ante jam aliquoties ad perihelium acceſſerit, niſi terra eo tempore in iisdem fere orbitæ ſuæ locis ſit verſata, longe alia ſpecie hunc eundem cometam conſpici debuiſſe, ut ex terra viſus nullo modo pro eodem haberi potuerit. Tum vero ad quod maxime eſt attendendum, hic inſignis cometa ſæpius ad ſolem reverti poterit, incolis terræ prorſus inconſpicuus. In hoc enim Cometa idem, quod de cometa anni 1742 notavi, uſu venit, quod diſparuerit, fere antequam ejus diſtantia a terra diſtantiam ſolis ſuperaſſet. Quare cum terra eo tempore, quo iſte cometa ante redierit, in orbita ſua ita ſita eſſe poſſit, ut is perpetuo longius quam ſol a terra fuerit remotus, nequidem obſervari potuit; unde eo minus eſt mirandum, hunc cometam ante nunquam eſſe animadverſum, quantum quidem ex obſervationibus ſufficienti ſtudio inſtitutis colligere licet. Confir-

matur

matur itaque maxime illa conjectura, quam ante feci, nullum cometam nobis eſſe conſpicuum, niſi terræ ſit propior quam ſol, ſicque ſine dubio complures cometæ ad ſolem in orbitis ſuis ſæpius revertuntur, aſpectum noſtrum prorſus fugientes. Et hanc ob rem numerus cometarum ſyſtema ſolare exornantium multo erit major, quam ſuſpicamur; utrum autem quandoque cometas, qui ad aliarum ſtellarum fixarum *ſyſtemata* pertineant, ſpectare nobis liceat; hoc nullo modo affirmari poterit, cum eorum diſtantia plus quam millies ſuperare deberet eam, in qua cometæ noſtri ſyſtematis ſe demum ſpectandos exhibeant.

Quoniam ceterum cometæ ſunt corpora non admodum ingentia, & terram in perigæo celerrime præterire ſolent, effectus eorum ratione attractionis in perturbatione motus terræ non minimus eſſe nequit: niſi forte proxime ad terram accedant; quod quidem fieri non poteſt, niſi dum in alterutro nodo verſantes per ipſam terræ ſemitam tranſeant, ibique terram offendant; unde non ſolum ob attractionem mutuam, ſed etiam a mutuo conflictu effectus maxime funeſti reſultare deberent. Quod autem in majoribus diſtantiis motum terræ vix perturbare valeant, ratio eſt, quia vis ſolis, qua terra in orbita ſua retinetur, incomparabiliter eſt major, quam illa vis, quæ unquam a cometa proficiſci poſſet; ita ut vis cometæ a vi ſolis longiſſime ſuperetur ac penitus abſorbeatur. Hoc vero tantum eſt tenendum de viribus cometarum, quarum directiones in planum eclipticæ incidunt; aliter enim comparata eſt ratio earum viri-

um, quarum directio ad planum eclipticæ eſt normalis. Cum enim hæ vires a vi ſolis non afficiantur, nihil impedit, quominus effectum ſuum producant; qui dum terram de plano eclipticæ vel boream vel auſtrum verſus retrahere nituntur, [illegible] orbitam terræ in aliud planum deducent; unde cum axis terræ non afficiatur, obliquitas eclipticæ atque poſitio punctorum æquinoctialium & ſolſtitialium immutabitur, qui effectus eo magis erit ſenſibilis, quo propius cometa ad terram acceſſerit, ſimulque quo major eo tempore fuerit ejus latitudo. Hinc non dubitarim mutabilitatem obliquitatis eclipticæ, quæ nunc quidem extra dubium poſita videtur, pariter ac variationem, quam in ſtellarum fixarum longitudine & latitudine deprehendunt, ſoli cometarum actioni tanquam veræ cauſæ adſcribere; facile autem erit in poſterum hanc conjecturam per obſervationes vel confirmare vel refellere, cum ſi hypotheſis hæc eſſet vera, iſtæ mutationes ſubito poſt apparitionem cujusque cometæ contingere deberent. Quod quidem ad cometam anni 1742 attinet, ejus actione obliquitas eclipticæ aliquantum augeri debuiſſet, quamobrem Aſtronomi ſunt rogandi, ut quam primum fieri licet, obliquitatem eclipticæ nunc quidem omni ſtudio inveſtigent, ut intelligi poſſit, utrum ea major reperiatur, quam ante hujus cometæ apparitionem eſt inventa, an ſecus. Generatim vero cometarum, qui in perigæo ſatis prope ad terram accedunt, ſimulque notabilem latitudinem habent, effectus ita erit comparatus; ut ſequens tabella declarat:

Si cometæ Latitudo fuerit borealis &

Locus ſolis ſit in ♈. Puncta æquinoctialia non afficientur, obliquitas eclipticæ autem augebitur.

Locus ſolis ſit in ♋. Puncta æquinoctialia promovebuntur, obliquitas eclipticæ non mutabitur.

Locus ſolis ſit in ♎. Puncta æquinoctialia non afficientur, obliquitas eclipticæ diminuetur.

Locus ſolis in ♑. Puncta æquinoctialia regredientur, obliquitas eclipticæ non afficietur.

Sin cometæ latitudo fuerit auſtralis, effectus iſti erunt contrarii. Probe autem iſta æquinoctiorum variatio diſtingui debet a præceſſione æquinoctiorum ſatis perſpecta, quæ non a mutabilitate eclipticæ, ſed a mutatione ipſius axis terræ proficiſcitur, qui a cometis non variatur. Hinc autem recepta Aſtronomorum regula, qua ſtatuunt, puncta æquinoctialia quotannis 50″ regredi, non parum perturbabitur. Quod autem ab antiquiſſimis temporibus obliquitas eclipticæ ſatis ſenſibiliter ſit minuta, concludendum eſt, vel plures cometas cum latitudine boreali, dum ſol in ſignis borealibus verſabatur, vel cum latitudine auſtrali, dum ſol in ſignis auſtralibus verſabatur, ad terram acceſſiſſe: horumque effectum prævaluiſſe.

Inveſtigatio
Orbitæ Cometæ
qui A. 1744. apparuit.

Obſervationes, ex quibus orbitam hujus cometæ ſum determinaturus, mecum Pariſiis ſunt communicatæ, quæ cum videantur omni cura inſtitutæ, perquam ſunt idoneæ, ad quas computus ſecundum methodum meam accommodetur. Prima quidem obſervatio facta eſt Lauſannæ jam die 13 Decembr. anni præterlapſi, quæ cum omnium, quas quidem adhuc mihi videre contigit, prima ſit, merito in hunc finem adhibetur. Hæc autem cum reliquis, quas accepi, ad meridianum Pariſinum revocata ſequenti modo ſe habebat.

Pariſiis Temp. appar.	Long. Cometæ	Lat. Com. bor.
1743 Dec. $13^d, 8^h, 1', 45''$	♈ $28°, 26', 13''$	$15°, 11', 0''$
1744 Jan. 3, 5, 27, 40	♈ 14, 11, 10	17, 32, 50
Jan. 7, 5, 1, 43	♈ 12, 3, 10	17, 51, 30
Jan. 18, 7, 2, 0	♈ 6, 57, 15	18, 37, 5

Quoniam igitur ad calculum inſtituendum mihi opus eſt tribus obſervationibus, inter quas temporum intervalla non nimis ſint inter ſe inæqualia, ex his elegi primam ſecundam & quartam, omittendo tertiam utpote ſecundæ nimis vicinam. Quia enim longitudo hujus cometæ hoc temporis intervallo parum fuit mutata, præſtat obſervationes aliquantum a ſe invicem remotas adhibere, quam nimis vicinas. Reductis ergo his obſervationibus ad tempus medium, ſupputatisque pro earum mo-

momentis terræ ſolisve locis una cum diſtantiis ſolis a terra, ſequentia prodierunt data:

Ordo Obſ.	Berol. temp. medio	Longit. Comet.	Latitudo Com.
I.	1743 Dec. $13^d, 8^h, 40'$	$0^s, 28°, 26', 13''$	$15°, 11', 0''$
II.	1744 Jan. 3, 6, 17	0, 14, 11, 10	17, 32, 50
III.	Jan. 18, 7, 57	0, 6, 57, 15	18, 37, 5

Temp. Obſ.	Locus ſolis	Log. diſt. ☉ a Terra
I.	$8^s, 21°, 30', 14''$	4, 9 9 2 9 0 3
II.	9, 12, 48, 18	4, 9 9 2 7 2 1
III.	9, 28, 9, 37	4, 9 9 3 0 3 2

Temporis ergo intervalla inter primam & ſecundam itemque ſecundam & tertiam obſervationem erunt

Inter I & II.	$20^d, 21^h, 37'$
II & III.	15, 1, 40

Exprimantur horæ cum minutis in partibus diei decimalibus, ut obtineantur valores pro litteris α & β, quæ in calculum ingrediuntur, erit

$$\alpha = 20,9008 \quad | \quad l\,\alpha = 1,320136$$
$$\beta = 15,0694 \quad | \quad l\,\beta = 1,178096$$

Delineetur jam figura his obſervationibus conveniens, ſitque tabula planum eclipticæ repræſentante, S locus ſolis, f, g, h, loca terræ in ſua orbita momentis trium obſervationum; ſit porro $f\zeta$ longitudo cometæ in obſervatione prima, $g\eta$ in ſecunda & $h\theta$ in tertia. Ductisque his lineis, quarum prima & tertia ſe in k, prima autem & ſecunda ſe in m, & ſecunda cum tertia in q interſecet, erunt cum lineæ tum anguli: Fig. 9.

49837

$l\,S\,f = 4,\ 992903$	$S\,f\,\zeta = 126^\circ,\ 55',\ 59''$
$l\,S\,g = 4,\ 992721$	$S\,g\,\eta = 91,\ 22,\ 52$
$l\,S\,h = 4,\ 993032$	$S\,h\,\theta = 68,\ 47,\ 38$
$f\,S\,g = 21^\circ, 18', 4''$	$f\,k\,\theta = 21,\ 28,\ 58$
$g\,S\,h = 15, 21, 19$	$f\,m\,g = 14,\ 15,\ 3$
$f\,S\,h = 36, 39, 28$	$g\,\eta\,h = 7,\ 13,\ 55.$

His præparatis, ſi nota eſſet cometæ a terra diſtantia vera in obſervatione media, orbita cometæ poſſet determinari, deficiente autem hac notitia, plures diſtantiæ fingi debebunt, ex ſingulisque orbita cometæ, quam eſſet habiturus, deduci, ut pateat, quænam hypotheſis proxime ad parabolam manuducat, quoniam tuto aſſumere poſſumus, veram cometæ orbitam parum a parabola diſcrepare. Siquis autem de hoc dubitet, poterit cometæ obſervatio quædam quarta ab aſſumtis ſatis remota in ſubſidium vocari, atque ex orbitis, quas ſingulæ hypotheſes ſuppeditaverunt, ad hoc tempus locus cometæ ſupputari, quo pateat, quænam earum proxime cum obſervatione hac quarta conveniat. In hunc finem adhibui obſervationem hic in Obſervatorio Academico factam die 18. Febr. cum cometa ſtellæ Marchab eſſet proximus, hinc autem collecta fuit.

Temp. medio Berol.	Longitudo Cometæ	Latitudo Bor.
A. 1744. Febr. $18^h, 6^h, 43''$	$11^s,\ 19^\circ,\ 57',\ 0''$	$19^\circ,\ 10',\ 56''$

Hoc autem tempore erat

Locus Solis $10^s, 29^\circ, 30', 40''$

& log. diſt. ſolis a terra $= 4,\ 995309.$

Pluribus igitur factis hypotheſibus circa diſtantiam cometæ a

terra

terra in obſervatione media, eam maxime huic obſervationi ſatisfacere deprehendi, quæ ſimul proxime parabolam exhiberet. Primum quidem ſuſpicatus ſum, cum iſte cometa tantopere fulgeret, eum a nobis non admodum fuiſſe remotum, ideoque initio hanc diſtantiam finxi, 20000 & 30000 poſita media terræ a ſole diſtantia 100000, hinc autem orbita prodiit elliptica parum excentrica, non multum a circulo diſcrepans; unde hæ hypotheſes nimium a veritate abhorrebant. Majores igitur valores tribui huic diſtantiæ, neque prius orbita in hyperbolam abire coeperat, quam iſtam diſtantiam = 110000 poſuiſſem; limites autem, intra quos ea diſtantiæ magnitudo, quæ obſervationi quartæ ſatisfaceret, inveni 101000 & 106000; ſicque præter expectationem diſtantia cometæ a terra multo major evaſit, quam initio putaveram. Cometa igitur a nobis fere æque erat remotus ac ſol, & cum ejus diameter apparens æſtimaretur unius circiter minuti primi, ejus diameter vera ad diametrum terræ rationem fere habebat triplam.

Sit igitur G verus cometæ locus in obſervatione ſecunda, ex quo ad eclipticam demittatur perpendiculum $G\eta$, & quoniam diſtantia Gg tanquam cognita aſſumitur, ob angulum $Gg\eta$ latitudini obſervatæ, $17^\circ 32', 50''$ æqualem; prodibit $G\eta = Gg. \sin Gg\eta$ & $g\eta = Gg. \cos Gg\eta$. Factis ergo binis memoratis hypotheſibus, calculus ſequenti modo inſtituatur:

Hypoth.

		Hypoth. A.	Hypoth. B.
G*g*	=	101000	106000
l G*g*	=	5, 004321	5, 025206
add. { *l* ſin G*g*η	=	9, 479275	9, 4792[illegible]
add. { *l* coſ. G*g*η	=	9, 979306	9, 979[illegible]
l Gη	=	4, 483596	4, 50458[illegible]
*l g*η	=	4, 983627	5, 004612

Ducatur nunc ex ſole recta Sη, & cum in triangulo S*g*η dentur latera S*g*, *g*η cum angulo intercepto S*g*η = 91°, 22°, 52,″ erit angulorum reliquorum ſumma = 88 , 37 , 8, & ſemiſumma = 44°, 18′, 34″ unde per trigonometriam reliqui anguli reperiuntur, quibus inventis erit $S\eta = \frac{Sg.\ \text{ſin}\ Sg\eta}{\text{ſin}\ S\eta g}$.

		A.	B.
A *l* S *g*	=	4, 992721	4, 992721
ſubtr. *l g* η	=	4, 983627	5, 004612
l tang.		10, 009094	− 10, 011891
anguli.		45°, 36′, −½″	− 45°, 47′ 3″.6
ſubtrahatur		45°	45°
reſt. ang.		0, 36, −½	0, 47, 3. 6
l tang.	=	8, 019943	8, 136401
l tang. ½ ſummæ.		9, 989530	9, 989530
l tang. ½ diff.		8, 009473	8, 125931
½ diff.	=	0, 35, 8	0, 45, 56
½ ſumma	=	44, 18, 34	44, 18, 34
S η *g*	=	44, 53, 42	43, 32, 38
g S η	=	43, 43, 26	45, 4, 30

Porro

	A	B
Porro eſt $l\,Sg$ =	4, 992721	4, 992721
l ſin $Sg\eta$ =	9, 999874	9, 999874
	4, 992595	4, 992595
ſubtr. l ſin $S\eta g$ =	9, 848687	9, 838162
$l\,S\eta$ =	5, 143908	5, 154433
& $S\eta$ =	139287	142703

Quia nunc in triangulo $GS\eta$ ad η rectangulo dantur latera $S\eta$ & $G\eta$, erit tang $GS\eta = \frac{G\eta}{S\eta}$ & $GS = \frac{S\eta}{\text{coſ}\, GS\eta}$.

	A	B
$l\,G\eta$ =	4, 483596	4, 504581
ſubtr. $l\,S\eta$ =	5, 143908	5, 154433
l tang $GS\eta$ =	9, 339688	9, 350148
II. Lat. hel. $GS\eta$ =	12, 19, 55	12, 37, 23
ſubtr. l coſ $GS\eta$ =	9, 989861	9, 989374
a $l\,S\eta$ =	5, 143908	5, 154433
II. Diſt. Com. a ⊙ $l\,SG$ =	5, 154047	5, 165059

Invento puncto G, ſint F & H loca cometæ vera in prima ac tertia obſervatione, & ducta corda FH ſecet SG in O. Oſtendi autem in diſſertatione mea fore intervallum GO = $\frac{2a^3\,\text{ſin}\,\alpha\tau.\,\text{ſin}\,\beta\tau}{SG^2\,\text{coſ}(\alpha-\beta)\tau}$, ubi τ denotat motum terræ ſemidiurnum medium 29′, 34″. 098, ita ut in minutis ſecundis ſit τ = 1774, 098, & $l\tau$ = 3, 248977, hinc ob litterarum α & β valores ante datos reperiuntur anguli $\alpha\tau$ & $\beta\tau$ hoc modo:

$l\tau$ =	3, 248977	
add. $l\alpha$ =	1, 320163	
add. $l\beta$ =	1, 178096	
$l\alpha\tau$ =	4, 569140	
$l\beta\tau$ =	4, 427073	
unde $\alpha\tau$ =	37080″ =	10°, 18′, 0″
$\beta\tau$ =	26734 =	7, 25, 34
$(\alpha-\beta)\tau$ =	10346 =	2, 52, 26

Cum igitur c denotet diſtantiam ſolis a terra mediam = 100000, valor ſagittæ GO ſequenti modo definietur.

	A	B
add $l\,\text{ſin}\,\alpha\tau$ =	9, 252373	
add $l\,\text{ſin}\,\beta\tau$ =	9, 111422	
	8, 363795	
ſubtr. $l\,\text{coſ}\,(\alpha-\beta)\tau$ =	9, 999453	
	8, 364342	
add. $l\,2\,c^3$ =	15, 301030	
	13, 665372	13, 665372
ſubtr. 2 lSG =	10, 308094	10, 330118
lGO =	3, 357278	3, 335254
add. $l\,\text{coſ}$ GSη =	9, 989861	9, 989374
prodibit $l\eta o$ =	3, 347139	3, 324628
unde ηo =	2224	2112
ſubtr. ab Sη =	139287	142703
remanebit So =	137063	140591

demiſſo ſcilicet ex puncto O in planum eclipticæ perpendiculo Oo. Nunc ſecet recta Sη reliquas cometæ longitudines in μ & ν, ad quæ puncta invenienda in triangulo S$f\mu$ primum dantur:

lSf =

	A	B
$lSf =$	4, 992903	4, 992903
ang. $Sf\mu =$	126, 55, 59	126, 55, 59
cujus deinc.	53, 4, 1	53, 4, 1
Ob ang. $gS\eta =$	43, 43, 26	45, 4, 30
ſubtr. $fSg =$	21, 18, 4	21, 18, 4
remanebit $fS\mu =$	22, 25, 22	23, 46, 26
qui ablatus ab $Sfk =$	53, 4, 1	53, 4, 1
relinquet $S\mu f =$	30, 38, 39	29, 17, 35

Ob datos ergo omnes angulos erit $f\mu = \frac{Sf. \text{ſin } fS\mu}{\text{ſin } S\mu f}$ & $S\mu = \frac{Sf. \text{ſin } Sf\mu}{\text{ſin } S\mu f}$, unde calculus dabit

A $lSf =$	4, 992903	4, 992903
ſubtr. l ſin $S\mu f =$	9, 707318	9, 689554
	5, 285585	5, 303349
add. l ſin $fS\mu =$	9, 581424	9, 605443
add. l ſin $Sf\mu =$	9, 902730	9, 902730
$lf\mu =$	4, 867009	4, 908792
$lS\mu =$	5, 188315	5, 206079
Ergo $f\mu =$	73622	81057
$S\mu =$	154282	160723
ſubtr. $So =$	137063	140591
reſtabit $o\mu =$	17219	20132

ſimili modo in triangulo Sbv data reperiuntur.

$lSb =$	4, 993032	4, 993032
ang. $Sbv =$	68, 47, 38	68, 47, 38
Ejus deinceps:	111, 12, 22	111, 12, 22
Deinde ob $gS\eta =$	43, 43, 26	45, 4, 30
add. $gSb =$	15, 21, 19	15, 21, 19

 erit

	A	B
erit bSv =	59 , 4, 45	60, 25, 49
qui ablatus ab externo =	111, 12, 22	111, 12, 22
relinquit Svb =	52 , 7, 37	50, 46, 33

Ob datos ergo omnes angulos cum latere Sb erit $bv =$ $\frac{Sb.\ \text{ſin}\ bSv}{\text{ſin}\ Svb}$ & $Sv = \frac{Sb.\ \text{ſin}\ Sbv}{\text{ſin}\ Svb}$. Ergo

	A	B
A lSb =	4, 993032	4, 993032
ſubtr. l ſin Svb =	9, 897282	9, 889121
	5, 095750	5, 103911
add. { l ſin bSv =	9, 933425	9, 939397
add. { l ſin Sbv =	9, 969548	9, 969548
lbv =	5, 029175	5, 043308
lSv =	5, 065298	5, 073459
Ergo bv =	106949	110486
Sv =	116225	118429
ſubtr. ab So =	137063	140591
remanebit ov =	20838	22162

Nunc per punctum o duci debet linea recta $\zeta o \theta$, cujus partes ζo & θo ſint in ratione temporum $\alpha : \beta$. Producatur ergo ov usque ad i, ut ſit $oi : vo = \alpha : \beta$. ſeu $oi = \frac{\alpha}{\beta}.\ ov$, tum ducatur recta $i\zeta$ parallela ipſi bv, eritque recta $\zeta o \theta$ recta quæſita.

	A	B
Ad lov =	4, 318856	4, 345609
add. $l\alpha : \beta$ =	0, 142067	0, 142067
loi =	4, 460923	4, 487676
Ergo oi =	28902	30738
ſubtr. $o\mu$ =	17219	20132
remanebit μi =	11683	10606

Nunc

Nunc in triangulo $\mu\zeta i$ dantur omnes anguli cum latere μi

	A	B
$l\,\mu i =$	4, 067565	4, 025551
ang. $\zeta\mu i =$	30, 38, 39	29, 17, 35
$\mu\zeta i =$	21, 28, 58	21, 28, 58
$180 - \mu i\zeta =$	52, 7, 37	50, 46, 33.

Erit ergo $\zeta\mu = \frac{\mu i.\ \text{ſin}\ \mu i\zeta}{\text{ſin}\ \mu\zeta i}$ & $\zeta i = \frac{\mu i.\ \text{ſin}\ \zeta\mu i}{\text{ſin}\ \mu\zeta i}$.

A $l\,\mu i =$	4, 067565	4, 025551
ſubtr. l ſin $\mu\zeta i =$	9, 563743	9, 563743
	4, 503822	4, 461808
add. { l ſin $\mu i\zeta =$	9, 897282	9, 889121
add. { l ſin $\zeta\mu i =$	9, 707318	9, 689554
$l\,\zeta\mu =$	4, 401104	4, 350929
$l\,\zeta i =$	4, 211140	4, 151362
Ergo $\zeta\mu =$	25183	22435
add. $f\mu =$	73622	81057
erit $f\zeta =$	98805	103492

Jam in triangulo $oi\zeta$ dantur duo latera cum angulo intercepto $oi\zeta$

A $l\,oi =$	4, 460923	4, 487676
ſubtr. $l\,\zeta i =$	4, 211140	4, 151362
l tang:	10, 249783	10, 336314
ang:	60, 38,13	65,15,3.8
ſubtr.	45	45
	15, 38,13	20,15,3.8
ſumma ang:	52, 7,37	50, 46,33
ſemi ſumma.	26, 3,48.5	25,23,16.5
l tang. ſemiſ:	9, 689401	9, 676306
l tang. ang.	9, 447002	9, 566955

	A	B
l tang ſemid:	9, 136403	9, 243261
ſemi diff.	7, 47, 43	9, 55, 52
ſemi ſumma	26, 3, 48	25, 23, 16
$o\zeta i =$	33, 51, 31	35, 19, 8
$\zeta o i =$	18, 16, 5	15, 27, 24

Porro eſt $o\zeta = \frac{oi. \text{ ſin } oi\zeta}{\text{ſin } oi\zeta}$ unde invenitur hoc modo

Ad $l oi =$	4, 460923	4, 487676
add. l ſin $oi\zeta =$	9, 897282	9, 889121
	4, 358205	4, 376797
ſubtr. l ſin $o\zeta i =$	9, 745968	9, 762022
$l o\zeta =$	4, 612237	4, 614775

ſi itaque corda $\theta\zeta$ producatur in n ob $b\theta n = o\zeta i$ erit

ang. $b\theta n =$	33, 51, 31	35, 19, 8

Nunc ob triangula $vo\theta$ & $io\zeta$ ſimilia erit $\theta v = \frac{\beta}{\alpha} . \zeta i$ & $\theta o = \frac{\beta}{\alpha} . o\zeta$, unde ſequenti modo invenientur

A $l\zeta i =$	4, 211140	4, 151362
ſubtr. $l\alpha:\beta =$	0, 142067	0, 142067
$l\theta v =$	4, 069073	4, 009295
$\theta v =$	11724	10216
ſubtr. a $bv =$	106949	110486
erit $b\theta =$	95225	100270
A $l o\zeta =$	4, 612237	4, 614775
ſubtr. $l\alpha:\beta$	0, 142067	0, 142067
$l\theta o =$	4, 470170	4, 472708
At eſt $o\zeta =$	40948	41188
& $o\theta =$	29524	29696
unde $\zeta\theta =$	70472	70884

Inventis

Inventis punctis ζ & θ ducantur rectæ Sζ & Sθ, ad quas inveniendas conſideretur primum triangulum S$f\zeta$, in quo ob data latera Sf & $f\zeta$ cum angulo intercepto S$f\zeta$ invenientur reliqua hoc modo.

ſubtr. lSf =	4, 992903	4, 992903
ab $lf\zeta$ =	4, 994779	5, 014906
l tang:	10, 001876	10, 022003
ang:	45, 7,25.5	46, 27,2.8
	45,	45,
	0, 7,25.5	1, 27,2.8
ſumma angul.	53,4, 1	53, 4,1
ſemi ſumma	26,32,0.5	26,32,0.5
l tang. ſemiſ.	9, 698371	9, 698371
l tang. ang.	7, 334519	8, 403571
l tang ſemid:	7, 032890	8, 101942
ſemi diff:	0, 3, 42	0,43, 28
ſemi ſumma	26, 32, 0	26, 32, 0
fSζ =	26, 35, 42	27, 15, 28
Sζf =	26, 28, 18	25, 48, 32

Jam erit $S\zeta = \frac{Sf.\ \text{ſin}\ Sf\zeta}{\text{ſin}\ S\zeta f}$, hinc fiet

lSf =	4, 992903	4, 992903
add. l S$f\zeta$ =	9, 902730	9, 902730
	4, 895633	4, 895633
ſubtr. l ſin Sζf =	9, 649096	9, 638859
lSζ =	5, 246537	5, 256774

Simili modo in triangulo S$b\theta$ ob data latera Sb, $b\theta$ cum angulo intercepto invenietur:

A lSb=

	A	B
A lSb =	4, 993032	4, 993032
ſubtr. $lb\theta$ =	4, 978751	5, 001171
l tang:	10, 014281	−10,008139
anguli =	45,56,30.7	45; 32,12.6
ſubtr. =	45	45
	0, 46,30.7	0, 32,12.6
ſumma ang:	111, 12,22	111, 12,22
ſemi ſumma =	55, 36, 11	55, 36, 11
l tang ſemi ſum: =	10, 164540	10, 164540
l tang. ang: =	8, 131313	7, 971728
l tang. ſemid: =	8, 295853	8, 136268
ſemidiff. =	1, 7, 56	0, 47, 3
ſemi ſumma =	55, 36, 11	55, 36, 11
$bS\theta$ =	54, 28, 15	56, 23, 14
$S\theta b$ =	56, 44, 7	54, 49, 8

Deinde cum ſit $S\theta = \frac{Sb.\ \text{ſin}\ Sb\theta}{\text{ſin}\ S\theta b}$ fiet

Ad lSb =	4, 993032	4, 993032
add. l ſin $Sb\theta$ =	9, 969548	9, 969548
	4, 962580	4, 962580
ſubtr. l ſin $S\theta b$ =	9, 922281	9, 912399
$lS\theta$ =	5, 040299	5, 050181

Ex his longitudo Cometæ heliocentrica ad tempora trium obſervationum determinabitur hoc modo.

Cum ſit $fS\zeta$ =	26, 35, 42	27, 15, 28
Long. terræ ad I.	2, 21,30,14	2, 21, 30,14
I. Long. Cometæ helioc.	1, 24,54,32	1, 24, 14,46
ſubtr. $gS\eta$ =	1, 13,43,26	1, 15, 4,30
a Long. terræ ad II.	3, 12,48,18	3, 12,48,18

II. Long

	A	B
II. Long. Cometæ helioc.	1, 29, 4,52	1, 27,43,48
Porro $b S \vartheta$ =	1, 24,28,15	1, 26,23,14
Long. terræ ad III	3, 28, 9,37	3, 28, 9,37
III. Long. Cometæ helioc. =	2, 3,41,22	2, 1,46,23
Hinc erit ang. $\zeta S \vartheta$ =	8,46,50	7,31,37

Cum igitur longitudo cometæ a prima ad tertiam obſervationem creſcat, manifeſtum eſt, verum cometæ motum fuiſſe directum, etiamſi ex terra retrogradus apparuerit.

Deinde etiam poſitio rectæ $\zeta\theta$ reſpectu linearum $S\zeta$ & $S\theta$ cognoſcetur, hac linea ultra θ in n producenda

Cum enim ſit $b\vartheta n$ =	33, 51, 31	35, 19, 8
ſubtr. ab $S\vartheta b$ =	56, 44, 7	54, 49, 8
erit $S\vartheta n$ =	22, 52, 36	19, 30, 0
ſubtr. $\vartheta S \zeta$ =	8, 46, 50	7, 31, 37
prodit ang. $S\zeta n$ =	14, 5, 46	11, 58, 23

Ex latitudinibus nunc geocentricis, inveniuntur perpendicula $F\zeta$ & $H\theta$, cum veris cometæ a terra diſtantiis Ff, Hb: Erit enim $F\zeta = f\zeta$. tang lat. I. $Ff = \frac{f\zeta}{\text{coſ. lat. I.}}$: atque $H\theta = b\theta$. tang lat. III. & $Hb = \frac{b\theta}{\text{coſ. lat. III.}}$.

$l f\zeta$ =	3, 994779	5, 014906
add. l tang. lat. I	9, 433580	9, 433580
ſubtr. l coſ lat. I	9, 984569	9, 984569
erit $l F\zeta$ =	4, 428359	4, 448486
& $l Ff$ =	5, 010210	5, 030337
Hinc fit $F\zeta$ =	26813	28085
I. Diſt. Com. a terra Ff =	102379	107235

	A	B
Porro $l b \vartheta$ =	4, 978751	5, 001171
add. l tang lat. III =	9, 527485	9, 527485
ſubtr. l coſ lat. III =	9, 976656	9, 976656
erit $l H \vartheta$ =	4, 506236	4, 529656
& $l H b$ =	5, 002095	5, 024515
Hinc fit $H \vartheta$ =	32080	33780
III. Diſt. Com. a terra $H b$ =	100484	105807
II. Diſt. Com. a terra $G g$ =	101000	106000

Cognitis nunc perpendiculis $H \vartheta$ & $F \zeta$, producatur recta HF, donec cum $\vartheta \zeta$ prolongata concurrat in N, eritque recta SN linea nodorum cometæ: erit autem $\frac{H \vartheta - F \zeta}{\zeta \vartheta}$ = tang $H N \vartheta$ & $\vartheta N = \frac{H \vartheta}{\text{tang } H N \vartheta}$: Ergo

A $H \vartheta$ =	32080	33780
ſubtr. $F \zeta$ =	26813	28085
erit $H \vartheta - F \zeta$ =	5267	5695
& $l (H \vartheta - F \zeta)$ =	3, 721563	3, 755494
ſubtr. $l \zeta \vartheta$ =	4, 848017	4, 850548
l tang $H N \vartheta$ =	8, 873546	8, 904946
ſubtr. a $l H \vartheta$ =	4, 506236	4, 528656
$l \vartheta N$ =	5, 632690	5, 623710

Conſideretur nunc triangulum $S \vartheta N$, in quo cum dentur latera $S \vartheta$, ϑN cum angulo intercepto, reperietur angulus $\vartheta S N$.

A $l \vartheta N$ =	4, 632690	5, 623710
ſubtr. $l S \vartheta$ =	5, 040299	5, 050181

l tang.

	A	B
l tang.	10, 592391	10, 573529
ang.	75, 39, 39	75 , 3 , 7
ſubtr.	45	45
	30, 39, 39	30 , 3 , 7
ſumma ang: Sϑ*n* =	22, 52, 36	19, 30 , 0
ſemi ſumma =	11, 26, 18	9, 45 , 0
l tang ſemiſ: =	9, 306063	9, 235102
l tang ang: =	9, 772930	9, 762348
l tang ſemid: =	9, 078993	8, 997450
Ergo ſemid:	6, 50, 23	5, 40, 38
ſemiſ:	11, 26, 18	9, 45, 0
Ang. ϑSN =	18, 16, 41	15, 25, 38
III. Long. helioc. puncti ϑ =	2, 3, 41,22	2, 1, 46,23
Longit. Nodi Aſcend: ☊ =	1, 15,24,41	1, 16,20,45

Demittatur nunc ex ϑ in lineam nodorum SN perpendiculum ϑP, ductaque HP erit angulus HPϑ æqualis inclinationi orbitæ cometæ ad Eclipticam; Fiet autem ϑP = Sϑ ſin ϑSN & tang. HPϑ = $\frac{H\vartheta}{\vartheta P}$.

Ad *l*Sϑ =	5, 040299	5, 050181
add. *l* ſin ϑSN =	9, 496415	9, 424905
*l*ϑP =	4, 536714	4, 475086
a *l*Hϑ =	4, 506236	4, 528656
l tang HPϑ =	9, 969522	10, 053570
Ergo ang. HPϑ = =	42, 59,28	48, 31,29
ideoque Inclinatio Orbitæ Cometæ ad Eclipticam =	42, 59, 28	48, 31, 29

Determinemus nunc quoque latitudines cometæ heliocentricas; quæ erunt: tang. $FS\zeta = \frac{F\zeta}{S\zeta}$ & tang $HS\vartheta = \frac{H\vartheta}{S\vartheta}$. Diſtantiæ vero cometæ a Sole erunt $SF = \frac{S\zeta}{\text{coſ}\, FS\zeta}$ & $SH = \frac{S\vartheta}{\text{coſ}\, HS\vartheta}$.

A $l F\zeta$ =	4, 428359	4, 448486
ſubtr. $l S\zeta$ =	5, 246537	5, 256774
l tang. $FS\zeta$ =	9, 181822	9, 191712
I. Lat. helioc. $FS\zeta$ =	8, 38, 32	8, 50, 18
A $l S\zeta$ =	5, 246537	5, 256774
ſubtr. l coſ $FS\zeta$ =	9, 995040	9, 994813
I. Diſt. Com. a Sole $l SF$ =	5, 251497	5, 261961
A $l H\vartheta$ =	4, 506236	4, 528656
ſubtr. $l S\vartheta$ =	5, 040299	5, 050181
l tang $HS\vartheta$ =	9, 465937	9, 478475
III. Lat. helioc. $HS\vartheta$ =	16, 17,51	16, 44,54
A $l S\vartheta$ =	5, 040299	5, 050181
ſubtr. l coſ $HS\vartheta$ =	9, 982188	9, 981175
III. Diſt. Com. a Sole. $l SH$ =	5, 058111	5, 069006

Determinemus nunc quoque elongationes cometæ heliocentricas ā nodo aſcendente ſeu linea SN, eritque coſ FSN = coſ $FS\zeta$. coſ ζSN & coſ HSN = coſ $HS\vartheta$. coſ ϑSN.

A ϑSN =	18,16,41	15,25,38
ſubtr. $\vartheta S\zeta$ =	8,46,50	7,31,37
remanet ζSN =	9,29,51	7,54, 1
l coſ $FS\zeta$ =	9, 995040	9, 994813
l coſ ζSN =	9, 994006	9, 995858

l coſ FSN =

	A	B
l coſ FSN =	9, 989046	9, 990671
Ergo ang. FSN =	12,48,52	11,49,59
l coſ HSϑ =	9, 982188	9, 981175
l coſ θSN =	9, 977516	9, 984063
l coſ HSN =	9, 959704	9, 965238
Ergo ang. HSN =	24,18,6	22,37,11
ſubtr. FSN =	12,48,52	11,49,59
Erit ang. FSH =	11,29,14	10,47,12

Quoniam igitur duo habemus puncta cometæ in orbita vera, nempe F & H, quorum diſtantiæ a foco S una cum angulo FSH ſunt cognitæ, hinc naturam orbitæ determinare poterimus. Quia vero SH < SF apparet cometam his obſervationum temporibus ad ſuum Perihelium acceſſiſſe. Sit igitur AHF vera orbita cometæ, circa focum S, quem ſol occupat, deſcripta, cujus vertex ſeu perihelium ſit in A. Sit diſtantia perihelii a ſole AS $= a$. applicata ex foco S ad axem normalis ſeu ſemilatus rectum SB $= b$, anomalia vera ſeu angulus ASH $= v$; tum vero ponantur cognita SH $= y$; SF $= z$; & angulus FSH $= \phi$. atque tempus, quo cometa ſpatium FH percurrit, in diebus expreſſum ſit $=$ T. Ex his primum inveniтur ſemilatus rectum $b = \frac{y^2 z^2}{4m^2 T^2} (\sin \phi)^2 + \frac{\sqrt{yz}}{3} (\sin \phi)^2$ Fig. 10.

exiſtente $m = 271989,735$ & $lm = 5,4345525$. hincque $l2m = 5,7355825$. Deinde fit tang $v = \cot \phi - \frac{(z-b)y}{(y-b)z \sin \phi}$.

tandemque $a = \frac{b y \cos v}{b - y + y \cos v}$. Ponatur poſtea $\frac{2a-b}{b} = n$;

P tangens-

tangensque anguli $\frac{1}{2}v = t$ fietque, ſi orbita proxime ad parabolam accedat, tempus quo cometa a loco H in perihelium A pervenit

$$\frac{[illegible]}{m\sqrt{b}}\left(\begin{array}{l} t + \frac{1}{3}t^3 - \frac{2}{5}nt^5 + \frac{3}{7}n^2t^7 - \frac{4}{9}n^3t^9 + \&c. \\ \quad + \frac{3}{5}n^2t^5 - \frac{4}{7}n^3t^7 + \frac{5}{9}n^4t^9 - \&c. \end{array}\right)$$

quod tempus erit expreſſum in diebus, dieique partibus decimalibus. Quia igitur tempus tertiæ obſervationis, quo cometa in puncto H hæſit, cognitum eſt, hinc momentum, quo per perihelium tranſiit cognoſcitur.

		A.	B.
Erit ergo $l\,y$	=	5, 058111	5, 069006
$l\,z$	=	5, 251497	5, 261961
$l\,y.z$	=	10, 309608	10, 330967
$T = \alpha + \beta$	=	35, 9702	
$l\,T$	=	1, 555941	
ϕ	=	11, 29, 14	10, 47, 12
l ſin ϕ	=	9, 299178	9, 272196
$l\,T^2$	=	3, 111882	
$l\,4.m^2$	=	11, 471165	
$l\,4m^2T^2$	=	14, 583047	
$l\,y^2z^2$	=	20, 619216	20, 661934
add. $2\,l$ ſin ϕ	=	8, 598356	8, 544392
		19, 217572	19, 206326
ſubtr. $l\,4m^2T^2$	=	14, 583047	14, 583047
l Partis prior.	=	4, 634525	4, 623279
$l\sqrt{yz}$	=	5, 154804	5, 165484
add. $2\,l$ ſin ϕ	=	8, 598356	8, 544392
		3, 753160	3, 709876
ſubtr. $l\,3$		0, 477121	0, 477121

	A	B
l part. poſt. =	3, 276039	3, 232755
Pars prior =	43105	42003
Pars poſt. =	1888	1709
b =	44993	43712
y =	114317	117221
z =	178442	182794
$y - b$ =	69324	73509
$z - b$ =	133449	139082
A *l* $(z-b)$ =	5, 125316	5, 143270
ſubtr. *l* $(y-b)$ =	4, 840884	4, 866341
	0, 284432	0, 276929
add. *l* $\frac{y}{z}$ =	9, 806614	9, 807045
	0, 091046	0, 083974
ſubtr. *l* ſin φ =	9, 299178	9, 272196
l part. ſubtr. =	0, 791868	0, 811778
Pars ſubtr. =	6, 19253	6, 48303
a cot φ =	4, 92077	5, 24883
— tang *v* =	1, 27176	1, 23420
Ergo 180 — *v* =	51,49,18	50,59,3
& anomalia vera *v* =	128,10,42	129, 0, 57
ideoque ang. ASH =	4ˢ,8,10,42	4, 9, 0, 57
add. HSN =	24,18, 6	22, 37, 11
Diſt. Perihelii a Nodo ☊:	5, 2,28,48	5, 1,38,8
Diſt ☋ a perihelio =	27, 31, 12	28, 21, 52
l — coſ *v* =	9, 791067	9, 799021
l y =	5, 058111	5, 069006
l — *y* coſ *v* =	4, 849178	4, 868027
add. *l b* =	4, 653145	4, 640601
l — Numer.:	9, 502323	9, 508628

		A	B
$-y \cos v$	=	70661	73795
add. $y - b$	=	69324	73509
$-b + y - y \cos v$	=	139985	147304
l — Denom.	=	5, 146081	5, 168214
a l — Num.	=	9, 502323	9, 508628
$l\, a$	=	4, 356242	4, 340414
Diſt. Perih. a Sole a	=	22711	21898
hinc $2\, a$	=	45422	43796
b	=	44993	43712
$2\, a - b$	=	429	84
$l\,(2a - b)$	=	2, 632458	1, 924280
ſubtr. $l\, b$	=	4, 653145	4, 640601
$l\, n$	=	7, 979313	7, 283679
$l\, a\, a$	=	8, 712484	8, 680828
ſubtr. $l \sqrt{b}$	=	2, 326572	2, 320300
		6, 385912	6, 360528
ſubtr. $l m$	=	5, 434553	5, 434553
$l \frac{a\, a}{m \sqrt{b}}$	=	0, 951359	0, 925975
Ob v	=	128, 10, 42	129, 0, 57
erit $\frac{1}{2}\, v$	=	64, 5, 21	64, 30, 28
& $l\, t$	=	0, 313536	0, 321655
$l\, t^2$	=	0, 627072	0, 643310
$l\, t^3$	=	0, 940608	0, 964965
$l\, t^5$	=	1, 567680	1, 608275
$l\, t^7$	=	2, 194752	2, 251585
$l\, t^9$	=	2, 821824	2, 894895
$l\, n\, t^5$	=	9, 546993	8, 891954
$l n^2 t^5$	=	7, 526306	6, 175633

$l n^2 t^7 =$

		A	B
$l n^2 t^7$	=	8, 153378	6, 818943
$l n^3 t^7$	=	6, 132691	4, 102622
$l n^3 t^9$	=	6, 759763	4, 745932
Ergo t	=	2, 05843	2, 09727
$+ \frac{1}{3} t^3$	=	2, 90728	3, 07499
		4, 96571	5, 17226
ſubt. $\frac{2}{5} n t^5 + \frac{4}{7} n^3 t^7 + \frac{4}{9} n^3 t^9$	=	0, 14126	0, 03119
		4, 82445	5, 14107
add. $\frac{3}{5} n^2 t^5 + \frac{3}{7} n^2 t^7 +$ &c.	=	811	9
$t + \frac{1}{3} t^3 +$ &c.	=	4, 83256	5, 14116
$l\,(t + \frac{1}{3} t^3 +$ &c.	=	0, 684177	0, 711062
add. $l \frac{aa}{m \sqrt{b}}$	=	0, 951359	0, 925975
l temp.	=	1, 635536	1, 637037
Temp.	=	43, 205	43, 355
ſeu		$43^d, 4^h, 55'$	$43^d, 8^h, 31'$
At tertia Obſerv. Jan.		18, 7, 57	18, 7, 57
Cometa in Perih. Mart.		1, 12,52	1^d,16,28

Orbita ergo Cometæ ſequentibus ſex momentis determinabitur:

Pro hypoth. G g	=	101000	106000
1. Diſtantia Perihelii a Sole ſeu a	=	22711	21898
& $l\,a$	=	4, 356242	4, 340414
2. Semilatus rectum orbitæ ſeu b	=	44993	43712
& $l\,b$	=	4, 653145	4, 640601

	A	B
3. Cometa per perihelium tranſiit A. 1744 Menſe Martio Temp. medio Berol.	$1^d,12^h,52'$	$1^d,16^h,28'$
4. Diſtantia Perihelii a nodo aſcendente ☊ =	$152°,28',48''$	$151°,38',\ 8''$
Hinc a Perihelio ad nodum deſcendentem ☋ = eſt anomalia vera =	27, 31, 12	28, 21, 52
5. Longitudo heliocentrica Nodi aſcend.	$1^s,15°,24',41''$	$1^s,16°,20',45''$
longitudo heliocentrica nodi deſcendentis =	$7^s,15°,24',41''$	$7^s,16°,20',45''$
6. Inclinatio Orbitæ Cometæ ad Eclipticam	$42°,59',28''$	$48°,31',29''$

Mox autem apparebit veram Cometæ orbitam intra hos duos limites tam parum a ſe invicem diſcrepantes contineri.

Computetur ergo ex utroque limite locus cometæ ad tempus obſervationis factæ d. 18 Febr. quæ adhuc ante appulſum ad perihelium contigit. Quæratur igitur primo intervallum temporis inter tranſitum cometæ per perihelium & momentum obſervationis, idque in diebus, dieique partibus decimalibus exprimatur, quod deinde vocetur = T.

	A	B
Perihel. Cometæ Mart.	$1^d,12^h,52'$	$1^d,16^h,28'$
ſubtrahatur Febr.	18 , 6, 43	18 , 6, 43
	12 , 6 , 9	12 , 9, 45
Ergo eſt T =	12, 2562	12, 4062
& l T =	1, 088355	1, 093639

Ponatur

Ponatur anomalia vera huic tempori reſpondens $= v$, ſitque tang. $\frac{1}{2} v = t$, erit $T = \frac{aa}{m\sqrt{b}} \left(t + \frac{1}{3} t^3 \begin{matrix} - \frac{2}{5} n t^5 \\ + \frac{3}{5} n^2 t^5 \end{matrix} \text{ \&c.} \right)$ Quoniam orbita parum a parabola diſcrepat, quæratur ex tabula motus in parabola θ, ut ſit $\theta + \frac{1}{3} \theta^3 = \frac{m\sqrt{b}}{aa} . T$ ſeu areæ parabolicæ, quo valore ipſius θ invento erit

$$\theta + \frac{1}{3}\theta^3 = t + \frac{1}{3} t^3 \begin{matrix} - \frac{2}{5} n t^5 + \frac{3}{7} n^2 t^7 - \\ + \frac{3}{5} n^2 t^5 - \frac{4}{7} n^3 t^7 + \end{matrix} \text{ \&c.}$$

& quia t vehementer parum a θ diſcrepat, ponatur $t = \theta + q$ erit $o = q + \theta\theta q \begin{matrix} - \frac{2}{5} n \theta^5 + \frac{3}{7} n^2 \theta^7 - \\ + \frac{3}{5} n^2 \theta^5 - \frac{4}{7} n^3 \theta^7 + \end{matrix}$ &c. ideoque fiet proxime

$$q = \frac{(\frac{2}{5} n - \frac{3}{5} n^2)\theta^5 - (\frac{3}{7} n^2 - \frac{4}{7} n^3)\theta^7 + (\frac{4}{9} n^3 - \frac{5}{9} n^4)\theta^9 - \text{\&c.}}{1 + \theta\theta}$$

Ita ergo calculus inſtituatur:

		A	B
$A\, l\, T$	=	1, 088355	1, 093639
ſubtr. $l \frac{aa}{m\sqrt{b}}$	=	0, 951359	0, 925975
$l(\theta + \frac{1}{3}\theta^3)$	=	0, 136996	0, 167664
Ergo 2 A tang θ	=	91°,3′,20″	93°,41′,55″
& A tang θ	=	45°,31,40	46, 50, 57
& $l\theta$	=	0,008001	0, 028052
$l\theta^2$	=	0,016002	0, 056104
$l\theta^5$	=	0, 040005	0, 140260
$l n$	=	7, 979313	7, 283679
$l n \theta^5$	=	8, 019318	7, 423939
$l n^2 \theta^5$	=	5, 998631	4, 707618

		A	B
$l n^2 \theta^7$	=	6, 014633	4, 763722
$l n^3 \theta^7$	=	3, 977946	
$+ \frac{2}{5} n \theta^5$	=	0, 004182	0, 001062
$- \frac{2}{5} n^2 \theta^5$	=	59	3
		0, 004123	0, 001059
$- \frac{3}{7} n^2 \theta^7$	=	44	3
Numerator	=	0, 004079	0, 001056
θ^2	=	1, 037535	1, 137900
Denom $1 + \theta^2$.	=	2, 037535	2, 137900
l Numerat.	=	7, 610554	7, 023664
l denom.	=	0, 309104	0, 329987
$l\, q$	=	7, 301450	6, 693677
q	=	0, 002002	0, 000494
θ	=	1, 018595	1, 066725
t	=	1, 020597	1, 067219
Ergo $\frac{1}{2} v$	=	45, 35, 2	46, 51, 45
Anomalia vera v	=	91, 10, 4	93, 43, 30
Perihelium a Nodo aſcend.	=	152, 28, 48	151, 38, 8
Diſt. Cometæ a Nodo	=	61, 18, 44	57, 54, 38

Porro diſtantia cometæ a Sole eſt $= \frac{b}{1 + \frac{b-a}{a} \operatorname{coſ} v}$.

Ergo a b	=	44993	43712
ſubtr. a	=	22711	21898
$b - a$	=	22282	21814
$l\,(b-a)$	=	4, 347954	4, 338735
ſubtr. $l\, a$	=	4, 356242	4, 340414
erit $l \frac{(b-a)}{a}$	=	9, 991712	9, 998321
add. l—coſ v	=	8, 309196	8, 812697

$l\, a - b$

	A	B
$l\,\frac{a-b}{a}\cos v =$	8, 300908	8, 811018
$-\frac{b+a}{a}\cos v =$	0, 019994	0, 064717
Denomin.	0, 980005	0, 935282
$l\,b =$	4, 653145	4, 640601
l Denom. =	9, 991228	9, 970942
l Diſt. Com. a Sole =	4, 661917	4, 669659

Reſolvendum nunc eſt triangulum ſphæricum ☊ C c, in quo eſt C ☊ diſtantia cometæ a nodo aſcendente, & angulus ☊ inclinatio orbitæ ad Eclipticam. Erit vero ſin C c = ſin ☊ C. ſin ☊ & tang ☊ c = tang ☊ C. coſ ☊. Fig. 11

☊ C =	61, 18, 44	57, 54, 38
ang. ☊ =	42, 59, 28	48, 31, 29
l ſin ☊ C =	9, 943122	9, 927995
l ſin ☊ =	9, 833710	9, 874621
l tang ☊ C =	9, 776832	9, 802616
l coſ ☊ =	10, 261847	10, 202702
l tang ☊ C =	9, 864190	9, 821052
l ſin C c =	10, 126037	10, 023754
Ergo lat. helioc. C c =	36, 44, 20	39, 24, 10.
& ☊ C =	1ˢ, 23, 12, 0	1, 16, 34, 0
addatur longitudo ☊ =	1, 15, 24, 41	1, 16, 20, 45
Long. helioc. Cometæ =	3, 8, 36, 41	3, 2, 54, 45
Long. helioc. terræ =	4, 29, 30, 40	1, 29, 30, 40
Ang. TS c =	1, 20, 53, 59	1, 26, 35, 55
ſumma ang. =	129, 6, 1	123, 24, 5
ſemi ſumma =	64, 33, 0	61, 42, 2
l S C =	4, 661917	4, 669659
l ſin C S c =	9, 776824	9, 802615
l coſ C S c =	9, 903833	9, 888012

Fig. 12.

Q 3 l C c =

	A	B
l Cc =	4, 438741	4, 472274
l Sc =	4, 565750	4, 557671
a lST =	4, 995309	4, 995309
l tang: =	10, 429559	10, 437638
ang:	69,35,57	69,56,42
ſubtr.	45	45
	24,35,57	24,56,42
l tang:	9, 660692	9, 667583
l tang ½ ſummæ:	10, 322480	10, 268869
l tang ſemid:	9, 983172	9, 936452
ſemi diff:	43,53,25	40,49,25
ſemi ſumma:	64,33, 0	61,42, 2
ang. STc =	20,39,35	20,52,37
Addatur longitudo Solis	10,29,30,40	10,29,30,40
Longitudo Cometæ Geoc.	11,20,10,15	11,20,23,17

Cum igitur longitudo cometæ obſervata ſit: 11ſ, 19°, 57′, 0″ vera orbita extra hos limites cadere videtur, ita ut ſtatui deberet Gg = 96000. Videamus ergo quoque latitudinem, eſt autem Tc = $\frac{Sc \sin TSc}{\sin STc}$.

	A	B
l Sc =	4, 565750	4, 557671
ſub l ſin STc =	9, 547550	9, 551890
	5, 018200	5, 005781
add l ſin TSc =	9, 889887	9, 921600
l Tc =	4, 908087	4, 927381
a l Cc =	4, 438741	4, 472274
l tang lát:	9, 530654	9, 544893
Lat. Geocentrica:	18°,44′,50″	19°,19′,30″

Cum

Cum igitur latitudo geocentrica eſſet obſervata 19°, 10′, 56″, orbita vera intra limites hoſce contineri deberet: Videtur autem latitudini plus ſidi oportere, quam longitudini; ſin autem utrique æqualiter fidere velimus, aberrationesque in utramque æqualiter diſtribuere, tum hypotheſis A veram orbitam cometæ præbere eſſet cenſenda. Quoniam vero quoque error in obſervationibus tribus aſſumtis ineſſe poteſt, qui etiamſi ſit minimus, tamen notabile diſcrimen in orbitam inferat, plus affirmare non licet, quam hos duos limites proxime ad veritatem accedere.

Interim tamen hoc tuto concludi poſſe videtur, orbitam cometæ neque hyperbolam eſſe, neque parabolam, ſed ellipſin vehementer oblongam: unde ſtatum habebit tempus periodicum. Erit enim diſtantia aphelii a ſole $= \frac{ab}{2a-b}$, tum ſemi axis transverſus $= \frac{aa}{2a-b} = c$

	A	B
Ergo a $2la =$	8, 712484	4, 680828
ſubtr. $l(2a-b) =$	2, 632458	1, 924280
lc	6, 080026	6, 756548
$l\sqrt{c} =$	3, 040013	3, 378274
unde $lc\sqrt{c} =$	9, 120039	10, 134822
ſubtr. $lc\sqrt{c} =$	7, 500000	7, 500000
Hinc tempus periodicum Cometæ prodiret	1, 620039	2, 634822
annorum:	41, $\frac{88}{100}$	431, $\frac{34}{100}$

Cometa denique in ſuo perihelio propius ad ſolem acceſſit, quam Mercurius, dum in ſuo perihelio verſatur. Hoc enim tempore

tempore diſtantia mercurii a ſole eſt = 30740, & quia medium inter utramque hypotheſin ſumendo diſtantia Cometæ in perihelio erat circiter 22000, illa ad hanc rationem proxime habebit ut 7 ad 5.

Definiamus vero adhuc tempus, quo cometa per ſuum nodum deſcendentem tranſierit; quo tempore erat ejus anomalia vera.

	A	B
$v =$	27°,31′,12″	28°,21′,52″

Cum autem angulus $\frac{1}{2} v$ fiat ſatis parvus, valor ipſius $t + \frac{1}{3} t^3 +$ &c. proxime ex hypotheſi parabolæ reperietur, fietque.

$l\,(t + \frac{1}{3} t^3 + \&c.) =$	9, 397478	9, 411748
add $l \frac{aa}{m\sqrt{b}} =$	0, 951359	0, 925975
	0, 348837	0, 337723
Tempus a Perihelio in diebus =	2, 2327	2, 1764
ſeu =	$2^d, 5^h, 45'$	$2^d, 4^h, 13'$
Addatur temp. Perih. Mart.	1, 12, 52	1, 16, 28
Cometæ per nodum deſcendentem tranſiit A. 1744 Martio	$3^d, 18^h, 37$	$3^d, 20^h, 41'$

Quare cometa die quarto menſis martii circa ortum ſolis per eclipticam auſtrum verſus eſt transgreſſus; motus ergo circa ſolem fuit celerrimus, quia duobus diebus fere 30 gradus in orbita ſua abſolvit. Tempus quo Cometa per nodum aſcendentem tranſiit tam accurate definiri nequit, quia ob ingentem anomaliam veram 151°, minimus error in orbita ingens diſcrimen producere valet. Interim ex limite B colligitur cometa per nodum aſcendentem tranſiiſſe. A. 1743. Menſis Auguſti die ſeptimo.

Quan-

Quanquam orbita cometæ hoc modo inventa parum a veritate diſcrepat, tamen per easdem, quibus uſus ſum obſervationes multo accuratius poteſt determinari eo modo, quem expoſui in Miſcell. Berol. Volumine VII. ubi oſtendi, quemadmodum, ſi orbita cometæ jam fere ſit cognita, ea per obſervationes corrigi debeat. Fingamus ergo orbitam a veritate parum diſcrepantem parabolicam; quæ contineatur his conditionibus

	Orbita ficta	ſitque orbita vera
Perihel. a ſole	22000	$22000 - \alpha$
ratio *b* : *a*	2 : 1	$2 - \frac{\beta}{10000} : 1$
Com. in Perih. Mart.	$1^d, 6^h, 0'$	$1^d, 6^h, \gamma'$
Diſt. Perih. a ☊	151°,	$151°, \delta'$
Long. hel. ☊	1^s, 16°	$1^s, 16° - \varepsilon'$
Incl. Orbitæ	45°	$45° + \zeta$.

Ut jam valores litterarum α, β, γ, δ, ε & ζ determinem, ſex conſtituo hypotheſes, quarum quælibet unica conditione ab orbita ficta diſcrepet. ſintque:

Hyp. I	Hyp. II	Hyp. III	Hyp. IV	Hyp. V	Hyp. VI
22000	22000	22000	22000	21000	22000
2 : 1	2 : 1	2 : 1	2 : 1	2 : 1	$2 - \frac{50}{10000} : 1$
$1^d, 6^h$	$1^d, 6^h$,	$1^d, 6^h$	$1^d, 18^h$	$1^d, 6^h$	$1^d, 6^h$
151°	151°	152°	151°	151°	151°
1^s, 16°	1^s, 15°	1^s, 16°	1^s, 16°	1^s, 16°	1^s, 16°
50°	45°	45°	45°	45°	45°

His conſtitutis eligo quatuor obſervationes omni cura inſti-

tutas, ad earumque tempora ex ficta orbita atque ex hypotheſibus investigo longitudinem & latitudinem geocentricam cometæ; atque ex discrimine ſingularum hypotheſium ab orbita ficta colligi poterit locus cometæ, quem orbita vera eſſet datura; qui cum obſervato comparatus dabit æquationem. Quoniam vero tantum ſex opus eſt æquationibus, ex quatuor illis obſervationibus duas latitudines rejiciamus, quippe quæ per reliqua ſponte determinantur. Hoc modo cum abſolviſſem calculum ſatis moleſtum, quem ob prolixitatem hic prætermitto, ſex ſequentes æquationes ſum adeptus:

I. Ex longitudine Dec. 13^d, 8^h, $40'$ obſervata:

$$483\zeta + 9383\epsilon - 6366\delta - 46\gamma + 335\alpha + 4220\beta - 41000 = 0$$

II. Ex longitudine Jan. 3^d, 6^h, $17'$ obſervata

$$1550\zeta + 6116\epsilon - 3716\delta - 130\gamma + 179\alpha + 3620\beta - 124000 = 0$$

III. Ex latitudine Jan. 3^d, 6^h, $17'$ obſervata

$$1260\zeta - 1566\epsilon + 6866\delta - 88\gamma - 495\alpha - 1380\beta - 421000 = 0$$

IV. Ex longitudine Jan. 18^d, 7^h, $57'$ obſervata

$$1517\zeta + 3883\epsilon - 1233\delta - 188\gamma + 63\alpha + 2640\beta - 156000 = 0$$

V. Ex latitudine Jan. 18^d, 7^h, $57'$ obſervata

$$1257\zeta - 1566\epsilon + 5583\delta - 86\gamma - 459\alpha - 1100\beta - 378000 = 0$$

VI. Ex longitudine Febr. 18^d, 6^h, $43'$ obſervata

$$1140\zeta - 1817\epsilon + 1733\delta - 544\gamma - 250\alpha + 560\beta - 131000 = 0$$

Ex his ſex æquationibus oriuntur ſex ſequentes valores ipſius ζ.

$$0 = \zeta + 19{,}426\varepsilon - 13{,}180\delta - 0{,}0952\gamma + 0{,}6936\alpha + 8{,}737\beta - 84{,}886$$
$$0 = \zeta + 3{,}946\varepsilon - 2{,}397\delta - 0{,}0838\gamma + 0{,}1154\alpha + 2{,}335\beta - 80{,}000$$
$$0 = \zeta - 1{,}243\varepsilon + 5{,}449\delta - 0{,}0705\gamma - 0{,}3930\alpha - 1{,}100\beta - 334{,}127$$
$$0 = \zeta + 2{,}559\varepsilon - 9{,}813\delta - 0{,}1239\gamma + 0{,}0415\alpha + 1{,}740\beta - 102{,}834$$
$$0 = \zeta - 1{,}246\varepsilon + 4{,}442\delta - 0{,}0684\gamma - 0{,}3651\alpha - 0{,}875\beta - 300{,}716$$
$$0 = \zeta - 1{,}594\varepsilon + 1{,}520\delta - 0{,}4772\gamma - 0{,}2193\alpha + 0{,}291\beta - 114{,}912$$

ſubtrahantur ſingulæ æquationes a prima eritque

$$0 = 15{,}480\varepsilon - 10\,783\delta - 0{,}0114\gamma + 0{,}5782\alpha + 6{,}402\beta - 4{,}886$$
$$0 = 20{,}699\varepsilon - 18{,}629\delta - 0{,}0247\gamma + 1{,}0866\alpha + 9{,}837\beta + 249{,}241$$
$$0 = 16{,}867\varepsilon - 12{,}367\delta + 0{,}0287\gamma + 0{,}6521\alpha + 6{,}997\beta + 17{,}948$$
$$0 = 20{,}672\varepsilon - 17{,}622\delta - 0{,}0268\gamma + 1{,}0587\alpha + 9{,}612\beta + 215{,}830$$
$$0 = 21{,}020\varepsilon - 14{,}700\delta + 0{,}3820\gamma + 0{,}9189\alpha + 8{,}246\beta + 30{,}026$$

hinc oriuntur quinque valores pro ε,

$$0 = \varepsilon - 0{,}6966\delta - 0{,}0007\gamma + 0{,}0373\alpha + 0{,}4135\beta - 0{,}3156$$
$$0 = \varepsilon - 0{,}9013\delta - 0{,}0012\gamma + 0{,}0526\alpha + 0{,}4776\beta + 12{,}0587$$
$$0 = \varepsilon - 0{,}7332\delta + 0{,}0017\gamma + 0{,}0387\alpha + 0{,}4148\beta + 1{,}0641$$
$$0 = \varepsilon - 0{,}8525\delta - 0{,}0013\gamma + 0{,}0512\alpha + 0{,}4650\beta + 10{,}4410$$
$$0 = \varepsilon - 0{,}6993\delta + 0{,}0182\gamma + 0{,}0437\alpha + 0{,}3923\beta + 1{,}4284$$

Subtrahantur ab ultimo omnes reliqui, atque orientur quatuor ſequentes æquationes.

$$0 = 0{,}0189\gamma - 0{,}0027\delta + 0{,}0064\alpha - 0{,}0212\beta + 1{,}7440$$
$$0 = 0{,}0194\gamma + 0{,}2020\delta - 0{,}0089\alpha - 0{,}0853\beta - 10{,}6303$$
$$0 = 0{,}0165\gamma + 0{,}0339\delta + 0{,}0050\alpha - 0{,}0225\beta + 0{,}3643$$
$$0 = 0{,}0195\gamma + 0{,}1532\delta - 0{,}0075\alpha - 0{,}0727\beta - 9{,}0126$$

 inveni-

invenientur ergo quatuor valores pro γ.

$$0 = \gamma - 0{,}1428\,\delta + 0{,}3386\,\alpha - 1{,}1217\,\beta + 92{,}275$$
$$0 = \gamma + 10{,}4124\,\delta - 0{,}4587\,\alpha - 4{,}4890\,\beta - 547{,}950$$
$$0 = \gamma + 0{,}2054\,\delta - 0{,}3030\,\alpha - 1{,}3636\,\beta + 22{,}078$$
$$0 = \gamma + 7{,}8564\,\delta - 0{,}3846\,\alpha - 3{,}7282\,\beta - 162{,}485$$

Subtrahantur prima ac tertia a ſecunda itemque prima a quarta.

$$0 = 10{,}5552\,\delta - 0{,}7973\,\alpha - 3{,}3673\,\beta - 640{,}225$$
$$0 = 10{,}2070\,\delta - 0{,}7617\,\alpha - 3{,}1254\,\beta - 570{,}028$$
$$0 = 7{,}9992\,\delta - 0{,}7232\,\alpha - 2{,}6065\,\beta - 554{,}460$$

Unde tres valores ipſius δ deducuntur:

$$\delta = 0{,}07554\,\alpha + 0{,}31902\,\beta + 60{,}655$$
$$\delta = 0{,}07462\,\alpha + 0{,}30620\,\beta + 55{,}846$$
$$\delta = 0{,}09041\,\alpha + 0{,}32585\,\beta + 69{,}315$$

ſubtrahatur medius ab utroque, eritque:

$$0 = 0{,}00092\,\alpha + 0{,}01282\,\beta + 4{,}809$$
$$0 = 0{,}01579\,\alpha + 0{,}01965\,\beta + 13{,}469$$

hincque bini valores pro β naſcuntur:

$$\beta = -0{,}07176\,\alpha - 375{,}118$$
$$\beta = -0{,}80356\,\alpha - 685{,}492$$

ſubtrahatur poſterior a priori eritque

$$0{,}73180\,\alpha + 310{,}374 = 0$$

Hinc

unde	$\alpha = - 424,125$	$l - \alpha = 2,627493$
hincque	$\beta = - 344,680$	$l - \beta = 2,537416$
	$\delta = - 81,345$	$l - \delta = 1,910331$
	$\gamma = - 346,889$	$l - \gamma = 2,540190$
	$\varepsilon = + 101,755$	$l + \varepsilon = 2,007555$
	$\zeta = + 311,44$	$l + \zeta = 2,493374$

Hinc ergo ob β negativum fiet orbita cometæ hyperbola ſequentibus ſex conditionibus determinata.

Diſtantia perihelii a Sole a =	22424
ratio $b : a$ =	$2 + \frac{344}{10000} : 1$
Ergo ſemilatus rectum b =	45619
Cometa per Perihelium tranſiit A. 1744. Mart.	$1^d, 0^h, 14'$
Diſt. Perihelii a ☊	$149^\circ, 39'$
Long. helioc. Nodi ☊	$1^s, 17^\circ, 41'$
Inclinatio orbitæ	$50^\circ, 11'$

Patet autem has determinationes maxime pendere a bonitate obſervationum, in quibus ſi vel minimum eſſet aberratum, orbita, quæ hic prodiit hyperbola, facile in ellipſin transmutari potuiſſet. Tum vero calculus quoque ſumma cura inſtitui debet, ita ut in locis, quæ ſingulæ hypotheſes præbent, ne minuta quidem ſecunda negligantur. Tantum laborem autem, niſi in obſervationibus tantam fiduciam collocare liceat, ſuſcipere operæ non eſt pretium. Quare mihi quidem methodum oſtendiſſe ſufficiat, cujus ope, ſi ac-

curatiſſimæ obſervationes ſuppetant, veræ orbitæ natura inveſtigari queat; quod negotium aliis expediendum relinquo.

Quoniam ergo ob defectum plurium obſervationum ſatis exactarum circa orbitæ naturam, utrum ſit in ſe rediens an in infinitum excurrens, nihil concludere licet, finem huic diſſertationi imponam, poſtquam quædam de hujus cometæ curſu tam obſervato quam futuro monuero. Primum igitur iſte cometa neque eclipticam neque æquatorem trajeciſſe viſus eſt, ſed quamdiu apparuit tam latitudinem quam declinationem habuit borealem, interim tamen ſi ad ejus tempus periodicum ſpectemus, quod ſi ullum eſt, plurium ſeculorum eſſe debet. Semeſtre tantum ſpatium in hemiſphærio boreali eſt verſatus, reliquo vero tot annorum ſpatio perpetuo in regione cœli auſtrali eſt commoratus. Deinde a 7 die Auguſti, quo per nodum ascendentem tranſiit usque ad Februarii diem 25 ab ecliptica receſſit, hinc vero curſu ſatis celeri jam die Martii 4to per nodum deſcendentem eſt transgreſſus, quæ ingens anomalia certe cum nulla alia theoria, præter Neutonianam, conſiſtere poteſt. Sub finem ergo apparitionis ejus via apparens vehementer a circulo cœli maximo deflexit: unde manifeſtum eſt, hunc cometam non in plano per centrum terræ tranſeunte eſſe motum. Cum in nodo deſcendente ver-

ſaretur

ſaretur, ſoli quidem propior fuit quam Mercurius, ab hoc vero tam parum fuit remotus, ut, ſi vim attractivam pro ratione molis habuerit, notabilis perturbatio in motu mercurii oriri debuiſſet: hoc enim tempore longitudo Cometæ heliocentrica erat, in Scorpii gradu 15, Mercurius vero in ejusdem ſigni gradu 26 hærebat; atque corpus cometæ, ſi diametrum apparentem eo tempore, quo in diſtantia ſolis a nobis erat remotus, ſtatuamus 1′, plusquam tricies corpus terræ ſuperavit. Quamobrem operæ pretium erit inveſtigare, utrum mercurius adhuc in motu ſuo cum tabulis aſtronomicis conſentiat, an vero perturbationem a cometa ſit paſſus.

Poſt Februarii diem ultimum, quo cometa adhuc ante ſolis ortum erat conſpicuus, penitus evanuit, cum ob ſolis vicinitatem, tum ob diminutam ejus latitudinem borealem. Quia enim poſt diem Martii quartum in hemisphærium auſtrale proceſſit, ante ortum ſolis non amplius ſupra horizontem noſtrum aſcendit. Quibus autem in cœli locis poſtea verſetur, ex his inventis ſatis accurate indicari poterit. Sic Aprilis die 15 iterum erit directus, & apparebit in 8vo Arietis gradu cum latitudine auſtrali 30° fere, & cum diſtantia a terra futura ſit paulo major quam ſolis, incolis terræ auſtralibus adhuc erit conſpicuus; qui poſt quartum diem martii hunc cometam ante ſolis ortum eximio

ſplendore

ſplendore videre debuerunt. Quareſi in his regionibus commorarentur Aſtronomi, diu adhuc poſt nos eundem cometam obſervare potuiſſent. Atque ſi eum exquiſitis tubis proſequerentur, fortaſſe ultra menſem Julium conſpicere poſſent, namque primo die Julii ejus longitudo eſſe debebit ♈ 27°, cum latitudine auſtrali 48°, ſexto autem die ſeptembris longitudo iterum erit ♈ 3°, cum latitudine 53°, ejus vero diſtantia a terra ſe habebit ad diſtantiam ſolis ut 2½ ad 1, unde niſi per bona teleſcopia vix ſpectari poterit. Hujusmodi autem obſervationes in regionibus terræ auſtralibus factæ maxime eſſent optandæ, cum ex iis facile omnia, quæ adhuc circa ejus orbitæ cognitionem deſiderantur, deduci ac ſuppleri poſſent. Utinam hoc tempore in capite bonæ ſpei idoneus aſtronomiæ cultor commoraretur, a quo iſtud ſupplementum expectare liceret.

Addita-

Additamentum.

I.

Cum ſuperiorem diſſertationem abſolviſſem, atque orbitæ cometæ determinationem ex obſervationibus initio commemoratis deductam ad Academiam Regiam Pariſinam miſiſſem; Celeb. Caſſinus, ut deſiderio meo theoriam per obſervationes comprobandi ſatisfaceret, omnes quas fecerat, obſervationes mecum benevole communicavit. Sunt autem ſequentes.

Pariſiis tempore apparente					Longitudo Cometæ obſervata				Latitudo obſervata B.		
1743.	Dec.	21^d,	6^h,	58$''$	0^s,	22°,	23$'$,	0$''$	16°,	18$'$,	57$''$
		30,	5,	54	0,	16,	29,	38	17,	12,	55
1744.	Jan.	1,	5,	41	0,	15,	19,	35	17,	23,	23
		3,	5,	28	0,	14,	11,	18	17,	32,	39
		4,	5,	21	0,	13,	38,	11	17,	37,	27
		5,	5,	14	0,	13,	5,	57	17,	42,	16
		6,	5,	8	0,	12,	34,	44	17,	46,	20
		7,	5,	2	0,	12,	3,	12	17,	51,	23
		8,	4,	55	0,	11,	33,	8	17,	55,	50
		10,	9,	42	0,	10,	24,	34	18,	5,	14
		11,	9,	1	0,	9,	58,	40	18,	9,	35
		12,	9,	11	0,	9,	31,	15	18,	13,	26
		13,	7,	52	0,	9,	5,	30	18,	17,	4
		16,	8,	43	0,	7,	45,	15	18,	30,	26
		17,	7,	41	0,	7,	20,	58	18,	34,	22
		18,	7,	0	0,	6,	56,	46	18,	38,	2

Febr.	1,	7,	55	0,	1,	9,	54	19,	34,	0
	3,	7,	49	0,	0,	18,	26	19,	42,	53
	7,	7,	55	11,	28,	16,	22	19,	53,	54
	10,	7,	17	11,	26,	32,	1	19,	56,	23
	11,	5,	47	11,	25,	52,	41	19,	57,	35
	12,	5,	51	11,	25,	12,	45	19,	56,	4
	13,	5,	39	11,	24,	28,	25	19,	53,	15
	15,	6,	46	11,	22,	46,	47	19,	44,	15
	16,	6,	19	11,	21,	54,	54	19,	36,	0
	17,	6,	30	11,	20,	55,	51	19,	23,	0
	18,	6,	3	11,	19,	54,	0	19,	10,	30
	23,	5,	34	11,	13,	12,	44	16,	41,	3
	24,	5,	47	11,	11,	36,	30	15,	48,	4
	25,	5,	22	11,	9,	52,	46	14,	39,	7
	29,	18,	44	11,	2,	31,	59	6,	28,	21

2. Primo quidem intuitu noſtra hujus cometæ theoria his obſervationibus mirifice confirmatur. Cum enim cometa ab initio apparitionis usque ad diem 18mum Februarii latitudinem fere eandem tenuiſſet, atque etiam ſecundum longitudinem ſatis tarde eſſet motus, hinc quaſi ſubito curſum eclipticam verſus inflectit, ſimulque ſecundum longitudinem motu citatiori progreditur. Ita igitur facile perſpicitur, quomodo cometa jam ante quartum Diem Martii ad eclipticam pervenire potuerit, uti theoria poſtulabat. Quin etiam ſi ad iſta obſervationum tempora ex orbita ante inventa cometæ loca ſupputentur, vix notabile diſcrimen deprehenditur: hincque adeo certe nobis concludere licet, affectiones orbitæ cometicæ detectas ad veritatem proxime accedere.

3. At

3. At vero quod caput rei eſt, & unde hoc additamentum eſt natum, iſtæ obſervationes adhiberi poſſunt, ad exiguas illas aberrationes, quibus orbita ſupra determinata adhuc laborat, emendandas, atque ad verum cometæ motum accuratiſſime definiendum. Hunc in finem ea ipſa methodus, quæ in hac diſſertatione eſt expoſita, hic quoque in uſu vocari poſſet; quia autem tria loca cometæ admodum vicina requirit, multitudo & inſigne obſervationum intervallum, quæ duæ res tamen maxime orbitis cometarum cognoſcendis inſerviunt, parum ſubſidii afferrent. Neque etiam hac methodo nunc amplius indigemus, cum orbita jam prope ſit nota, atque aliam dederimus methodum orbitas jam vero proximas per plures obſervationes corrigendi.

4. Interim tamen & hæc methodus, qua jam ſumus uſi ad curſum cometæ ex obſervationibus accuratius determinandum, non contemnendas habet difficultates. Primo enim nimis longum ac tædioſum calculum requirit, ac deinde multitudo incognitarum, quas ex aberrationibus definiri oportet non ſolum operationem reddit moleſtiſſimam, ſed etiam ob tot quantitates neglectas, minus certam. Quanquam enim iſtæ quantitates tam ſunt parvæ, ut ſingulæ ſine errore rejici poſſent, tamen eæ conjunctim ſumtæ, errorem notabilem procreare poſſunt. Hanc ob cauſam in aliam methodum inquiſivi, cujus ope orbita cometæ jam propemodum cognita non ſolum facilius & expeditius emendari queat, ſed quæ etiam pauciores in-

cognitas ad negotium abſolvendum requirat. Cum igitur hoc deſideratum aliquamdiu animo volviſſem, ſequentem methodum ſum aſſecutus, cujus ope ex datis tribus tantum cometæ obſervationibus quibuscunque orbita jam pæne cognita exactiſſime aſſignari poteſt.

5. Ne igitur ſex illa capita, quibus orbita cometæ determinatur, ſimul in computum inducantur, ex iis tantum aliquot tanquam cognita aſſumamus, unde ex dato quovis loco obſervato cometæ locus heliocentricus concludi queat. Hoc autem fieri poteſt ex poſitione lineæ nodorum, & inclinatione orbitæ ad eclipticam: his enim duobus tantum capitibus cognitis, ex quovis loco cometæ geocentrico ejus locus heliocentricus atque diſtantia a ſole definiri poteſt, quæ methodus pro planetis jam pridem eſt nota & paſſim explicata; interim tamen ob nimis parvam inclinationem orbitarum planetarum, niſi obſervationes ſint fere ad minuta ſecunda exactæ, tuto adhiberi nequit. Si enim inclinatio plane evaneſceret, tum hinc nihil prorſus concludi poſſet: ex quo facile colligitur, quo minor ſit orbitæ cujuspiam inclinatio, eo minus tuto hanc methodum adhiberi poſſe. Cum igitur cometarum orbitæ plerumque magnos angulos cum ecliptica conſtituant, ad eos hæc methodus maxime eſt accommodata, atque feliciſſimo ſucceſſu uſurpari poterit.

6. Si ergo aſſumamus poſitionem lineæ nodorum atque inclinationem orbitæ ad eclipticam eſſe datas, ex quovis come-

tæ loco obſervato ejus locum verum in cœlo aſſignare poterimus, ope ſequentis problematis. Quanquam autem hæ duæ res tantum prope ſunt cognitæ, ſolutio tamen iſtius problematis maximam afferet utilitatem, quoniam deinceps monſtrabo, quemadmodum per tres obſervationes eædem accuratius determinari queant.

Problema I.

Datis orbitæ cometæ interſectione cum ecliptica & ejus inclinatione ad eam, ad datum quodvis tempus, quo longitudo & latitudo cometæ geocentrica eſt obſervata, ejus locum verum heliocentricum cum ejus a ſole diſtantia invenire.

Solutio.

7. Computetur ad tempus propoſitum locus ſolis, ſitque diſtantia terræ a ſole $ST = c$. Denotet ſcilicet S ſolem, T terram, ſitque SN linea nodorum, cujus poſitio cum ſit data, dabitur angulus TSN qui ponatur $= s$. Deinde ducatur ſecundum longitudinem cometæ obſervatam recta TN lineam nodorum interſecans in N, eritque angulus STN differentia longitudinum ſolis & cometæ, & propterea datus. Poſito ergo angulo hoc $STN = t$, erit angulus $SNT = 180° - s - t$, qui ponatur $= n$. Hinc in triangulo STN ob omnes angulos cum latere $ST = c$ cognitos, reperietur $TN = \frac{ST \sin s}{\sin n}$ & $SN = \frac{ST \sin t}{\sin n}$. His præparatis ſit C locus cometæ Fig. 1

verus

verus in ſublimi poſitus, unde ad planum eclipticæ TSN demittatur perpendiculum Cc, eritque CTc latitudo cometæ geocentrica, quæ ſi ponatur $= p$, erit ang. CT$c = p$. Tum ſi ex c ad SN ducatur perpendicularis cP jungaturque recta CP, repræſentabit angulus CPc inclinationem orbitæ ad Eclipticam, ponatur ea $= i$, eritque CP$c = i$. Ex his conſiderationibus determinabitur punctum c. Poſita enim $c\mathrm{N} = x$; & $\mathrm{TN} = \frac{c \text{ ſin } s}{\text{ſin } n} = a$, erit $\mathrm{T}c = a - x$. Deinde in triangulo cNP fiet $c\mathrm{P} = x \text{ ſin } n$, & ex triangulo CP$c$ fiet $\mathrm{C}c = x \text{ ſin } n. \text{ tang } i$. At ex triangulo CT$c$ ſequitur $\mathrm{C}c = (a-x) \text{ tang } p$; quibus duobus valoribus æquatis reperitur $x = \frac{a \text{ tang } p}{\text{tang } p + \text{ſin } n. \text{tang } i} = c\mathrm{N}$ & $\mathrm{T}c = \frac{a \text{ ſin } n. \text{ tang } i}{\text{tang } p + \text{ſin } n. \text{tang } i}$ $= \frac{c \text{ ſin } s. \text{ tang } i}{\text{tang } p + \text{ſin } n. \text{tang } i}$ hincque $\mathrm{TC} = \frac{c \text{ ſin } s. \text{ tang } i}{\text{ſin } p + \text{ſin } n. \text{coſ } p \text{ tang } i}$ quæ eſt diſtantia cometæ a terra. Porro vero invento $c\mathrm{N} = x$, habebitur $\mathrm{PN} = x \text{ coſ } n$ & $c\mathrm{P} = x \text{ ſin } n$; cognosceturque $\mathrm{SP} = \mathrm{SN} - \mathrm{NP}$; unde oritur tang $c\mathrm{SN} = \frac{c\mathrm{P}}{\mathrm{SP}}$, cognitoque angulo cSN erit $\mathrm{S}c = \frac{c\mathrm{P}}{\text{ſin } c\mathrm{SN}} = \frac{\mathrm{SP}}{\text{coſ } c\mathrm{SN}}$. Tum vero erit tang $\mathrm{CS}c = \frac{\mathrm{C}c}{\mathrm{S}c}$; ſicque innoteſcit latitudo heliocentrica CSc, ex qua efficitur diſtantia cometæ a ſole $\mathrm{SC} = \frac{\mathrm{C}c}{\text{ſin } \mathrm{CS}c} = \frac{\mathrm{S}c}{\text{coſ } \mathrm{CS}c}$. Denique cum ſit $\frac{\mathrm{SP}}{\mathrm{SC}} =$ coſ

CSN;

C S N, hic angulus indicabit elongationem cometæ helio-centricam a nodo N, in ſua orbita. Cum igitur planum orbitæ cometæ detur, ex angulo N S C & recta S C definitur locus cometæ verus heliocentricus. Q. E. J.

Coroll. 1.

8. Cum ſit $a = \frac{c \sin s}{\sin n}$, erit $x = cN = \frac{c \sin s \operatorname{tang} p}{\sin n (\operatorname{tang} p + \sin n \operatorname{tang} i)}$ & $cP = \frac{c \sin s \operatorname{tang} p}{\operatorname{tang} p + \sin n \operatorname{tang} i}$; ideoque $CP = \frac{c \sin s \operatorname{tang} p : \cos i}{\operatorname{tang} p + \sin n . \operatorname{tang} i}$. Tum vero erit $PN = \frac{c \sin s \operatorname{tang} p \cot n}{\operatorname{tang} p + \sin n \operatorname{tang} i}$, qui valor ſubtractus ab $SN = \frac{c \sin t}{\sin n}$ relinquit $SP = \frac{c (\sin t \operatorname{tang} p + \sin n . \sin t . \operatorname{tang} i - \cos n . \sin s \operatorname{tang} p)}{\sin n (\operatorname{tang} p + \sin n \operatorname{tang} i)}$. At ob $\sin t = \sin n \cos s + \cos n . \sin s$, habebitur $SP = \frac{c (\cos s \operatorname{tang} p + \sin t \operatorname{tang} i)}{\operatorname{tang} p + \sin n \operatorname{tang} i}$.

Coroll. 2.

9. Quia igitur $\frac{CP}{SP}$ dat tangentem anguli CSN, erit

$$\operatorname{tang} CSN = \frac{\sin s \operatorname{tang} p}{\cos i \cos s \operatorname{tang} p + \sin t \sin i}.$$

Hinc ergo erit

$$\cot CSN = \frac{\cos i}{\operatorname{tang} s} + \frac{\sin t \sin i}{\sin s \operatorname{tang} p}.$$

Ex ſolis ergo angulis cognitis s, t, i, & p invenitur elongatio cometæ a nodo ſeu angulus CSN.

Coroll. 3.

Coroll. 3.

10. Invento angulo CSN, ponatur iſte angulus CSN $= m$, ut ſit $\cot m = \frac{\operatorname{coſ} i}{\operatorname{tang} s} + \frac{\operatorname{ſin} t \operatorname{ſin} i}{\operatorname{ſin} s \operatorname{tang} p}$, fiet $\frac{\operatorname{ſin} i}{\operatorname{tang} p}$ $= \frac{\operatorname{ſin} s \cot m}{\operatorname{ſin} t} - \frac{\operatorname{ſin} s \operatorname{coſ} i}{\operatorname{ſin} t \operatorname{tang} s}$. Cum igitur inventum ſit CP $= \frac{c \operatorname{ſin} s \operatorname{tang} p : \operatorname{coſ} i}{\operatorname{tang} p + \operatorname{ſin} n . \operatorname{tang} i} = \frac{c \operatorname{ſin} s}{\operatorname{coſ} i + \operatorname{ſin} n . \operatorname{ſin} i : \operatorname{tang} p}$, ſi hic valor ille loco $\frac{\operatorname{ſin} i}{\operatorname{tang} p}$ ſubſtituatur, erit CP $=$ $\frac{c \operatorname{ſin} t}{\operatorname{coſ} i \operatorname{coſ} n + \operatorname{ſin} n . \cot m}$.

Coroll. 4.

11. Quia porro $\frac{CP}{\operatorname{ſin} m}$ dat diſtantiam cometæ a ſole SC erit $CS = \frac{c \operatorname{ſin} t}{\operatorname{ſin} m . \operatorname{coſ} n . \operatorname{coſ} i + \operatorname{ſin} n . \operatorname{coſ} m}$. Hincque ex diſtantia terræ a ſole $ST = c$, & angulis m, n, i & t reperitur diſtantia cometæ a ſole CS. At vero ob $\operatorname{ſin} m . \operatorname{coſ} n =$ $\frac{1}{2} \operatorname{ſin} (m+n) + \frac{1}{2} \operatorname{ſin} (m-n)$; $\operatorname{coſ} m . \operatorname{ſin} n = \frac{1}{2} \operatorname{ſin} (m+n) - \frac{1}{2} \operatorname{ſin}$ $(m-n)$ atque $\frac{1 + \operatorname{coſ} i}{2} = (\operatorname{coſ} \tfrac{1}{2} i)^2$ & $\frac{1 - \operatorname{coſ} i}{2} = (\operatorname{ſin} \tfrac{1}{2} i)^2$ erit

$$CS = \frac{c \operatorname{ſin} t}{\operatorname{ſin} (m+n) (\operatorname{coſ} \tfrac{1}{2} i)^2 - \operatorname{ſin} (m-n) (\operatorname{ſin} \tfrac{1}{2} i)^2}.$$

Coroll. 5.

12. Commodiſſime ergo calculus inſtituetur, quærendo primum has quantitates:

cot.

$$\cot CSN = \frac{\operatorname{coſ} i}{\operatorname{tang} s} + \frac{\operatorname{ſin} t . \operatorname{ſin} i}{\operatorname{ſin} s \operatorname{tang} p}$$

$$\frac{ST}{CP} = \frac{\operatorname{coſ} i}{\operatorname{ſin} s} + \frac{\operatorname{ſin} n . \operatorname{ſin} i}{\operatorname{ſin} s . \operatorname{tang} p}$$

Quibus inventis erit $SC = \frac{CP.}{\operatorname{ſin} CSN}$.

Coroll. 6.

13. Inventa autem recta CP ſtatim innoteſcet diſtantia cometæ a terra CT. Cum enim ſit $\operatorname{ſin} p = \frac{Cc}{cT}$ & $\operatorname{ſin} i = \frac{Cc}{CP}$ erit $\frac{\operatorname{ſin} p}{\operatorname{ſin} i} = \frac{CP}{CT}$ ideoque $CT = \frac{CP. \operatorname{ſin} i}{\operatorname{ſin} p}$. Ad inſtitutum autem noſtrum diſtantia cometæ a terra non indigemus

Coroll. 7.

14. Si cometa in ipſa ecliptica obſervetur, ita ut ejus latitudo geocentrica p ſit nulla, fiet cot $CSN = \infty$ ideoque ipſe angulus CSN evaneſcet. Verſabitur ergo Cometa in ipſo puncto N, eritque ejus diſtantia a ſole $= \frac{c \operatorname{ſin} t}{\operatorname{ſin} n}$.

Coroll. 8.

15. Si terra in ipſa linea nodorum verſetur, fiet angulus $TSN = s = o$. Quo caſu & CP & angulus CSN evaneſcere videtur. At vero hoc caſu quoque fiet $\operatorname{tang} p + \operatorname{ſin} n . \operatorname{tang} i = o$, ita ut nihilominus lineæ cP & CP finitum valorem retineant. Quantæ autem ſint iſtæ lineæ una cum angulis hoc caſu definiri nequit. Quanquam autem hujusmodi obſervatio-

nes ad præſens inſtitutum inutiles videntur, tamen eæ inſervient, ſi poſitio lineæ nodorum jam fuerit cognita, ad inclinationem orbitæ accurate definiendam, erit enim tang $= -\frac{\text{tang } p}{\text{ſin } n}$: unde ad orbitas planetarum determinandas inſignis uſus redundat.

Coroll. 9.

16. Si fiat angulus $n = o$, quod evenit, ſi longitudo cometæ geocentrica congruat cum longitudine nodi heliocentrica, erunt ſinus angulorum s & t æquales, habebiturque diſtantia cometæ a terra $TC = \frac{c \text{ ſin } s \text{ tang } i}{\text{ſin } p}$, & cot $CSN = \frac{\text{coſ } i}{\text{tang } s} + \frac{\text{ſin } i}{\text{tang } p}$ atque $\frac{ST}{CP} = \frac{\text{coſ } i}{\text{ſin } s}$ ſeu $CP = \frac{\text{ſin } s}{\text{coſ } i} \cdot ST$.

Coroll. 17.

17. Si cometa in oppoſitione vel conjunctione ſolis ſecundum longitudinem fuerit obſervatus, erit ſin $t = o$: hoc ergo caſu habebitur cot. $CSN = \frac{\text{coſ } i}{\text{tang } s}$, atque $\frac{ST}{CP} = \frac{\text{coſ } i}{\text{ſin } s} + \frac{\text{ſin } i}{\text{tang } p}$ ob ſin n = ſin s.

Coroll. 11.

18. Si tandem inclinatio orbitæ cometæ ad eclipticam evaneſceret, ut eſſet $i = o$: tum neceſſe eſt ut quoque latitudo

titudo obſervata p ſit nulla, hocque ergo caſu ob fractionem $\frac{\text{ſin } i}{\text{tang } p}$ indefinitam nihil concludi poſſet.

Coroll. 12.

19. Niſi ergo tempore obſervationis locus terræ fuerit prope lineam nodorum, ex loco cometæ geocentrico obſervato ſemper ejus locus verus, hoc eſt elongatio ejus heliocentrica ab altero nodo una cum ejus diſtantia a ſole definiri poteſt.

20. Poſitis ergo linea nodorum & inclinatione orbitæ cognitis, ſi tria loca cometæ obſervata ad hunc calculum revocentur, invenientur tria loca cometæ vera in ſua orbita. Quam cum conſtet eſſe ſectionem conicam, cujus alter focus in ipſo centro ſolis ſit ſitus, ex iſtis tribus punctis inventis ſectio conica poterit determinari, quem in finem ſequens problema reſolvemus.

Problema II.

Datis tribus cometæ locis veris, una cum ejus diſtantiis a ſole, invenire ejus orbitam; hoc eſt poſitionem perihelii, ejus a ſole diſtantiam, atque quantitatem lateris recti.

Solutio.

21. Repræſentet planum tabulæ id ipſum planum in quo cometa movetur, in quo ſit S centrum ſolis & recta ☊ S ☋ linea nodorum ſeu interſectio hujus plani cum ecliptica. Sint F, G & H tria cometæ loca vera in ejus orbita, dabunturque primum diſtantiæ SF $= f$, SG $= g$ & SH $= h$; tum Fig. 2.

vero

vero per methodum præcedentem cognoſcuntur anguli F S ☋, G S ☋ & H S ☋; ex quibus ſit F S G $= \phi$ & F S H $= \Psi$. His datis contemplemur, quæ quæruntur, ſitque A orbitæ perihelium, A S C orbitæ axis, ad quem in F applicata normalis SB exhibebit ſemi-latus rectum. Ponatur AS $= a$, B S $= b$, ſitque angulus A S F $= v$, quæ eſt anomalia vera reſpondens loco F. Erit ergo A S G $= v + \phi$ & A S H $= v + \psi$. At ex ſectionum conicarum proprietatibus erit $f = \frac{ab}{a + (b-a)\,\text{coſ}\,v}$ unde ſit $a = \frac{bf\,\text{coſ}\,v}{b - f + f\,\text{coſ}\,v}$. Simili modo ex ſecundo loco G erit $a = \frac{bg\,\text{coſ}\,(v+\phi)}{b - g + g\,\text{coſ}\,(v+\phi)}$ atque ex tertio loco H habetur $a = \frac{bh\,\text{coſ}\,(v+\psi)}{b - h + h\,\text{coſ}\,(v+\psi)}$. Ex prima & ſecunda æquatione ſequitur $(b-g)f\,\text{coſ}\,v = (b-f)g\,\text{coſ}(v+\phi) = (b-f)$ $g\,(\text{coſ}\,v.\,\text{coſ}\,\phi - \text{ſin}\,v.\,\text{ſin}\,\phi)$; ac dividendo per coſ v proveniet $(b-g)f = (b-f)\,g\,\text{coſ}\,\phi - (b-f)\,g\,\text{ſin}\,\phi\,\text{tang}\,v$, unde obtinetur $\text{tang}\,v = \text{cot}\,\phi - \frac{f(b-g)}{g(b-f)}\,\text{coſec.}\,\phi = \frac{1}{\text{tang}\,\phi} - \frac{f(b-g)}{g(b-f)\,\text{ſin}\,\phi}$. Similiter prima æquatio cum tertia conjuncta dabit: tang v $= \frac{1}{\text{tang}\,\psi} - \frac{f(b-h)}{h(b-f)\,\text{ſin}\,\psi}$. Hisque binis ipſius tang v valoribus coæquatis erit $\frac{1}{\text{tang}\,\phi} - \frac{f(b-g)}{g(b-f)\,\text{ſin}\,\phi} = \frac{1}{\text{tang}\,\psi} - \frac{f(b-h)}{h(b-f)\,\text{ſin}\,\psi}$ ſeu

$\frac{b-f}{f \tang \phi} - \frac{b+g}{g \sin \phi} = \frac{b-f}{f \tang \psi} - \frac{b+b}{b \sin \psi}$: unde concluditur

$$b = \frac{\frac{1}{\tang \phi} - \frac{1}{\sin \phi} - \frac{1}{\tang \psi} + \frac{1}{\sin \psi}}{\frac{1}{f \tang \phi} - \frac{1}{g \sin \phi} - \frac{1}{f \tang \psi} + \frac{1}{b \sin \psi}},$$ quæ formula ad calculum ſatis eſt apta, faciliusque adhibebitur, quam conſtructiones geometricæ, quæ de hoc problemate paſſim inveniuntur. Invento igitur ſemi-latere recto b poſitio lineæ abſidum definietur ex angulo v, qui ex alterutra harum æquationum eruetur:

$$\tang v = \frac{1}{\tang \phi} - \frac{f(b-g)}{g(b-f)\sin \phi}; \quad \tang v = \frac{1}{\tang \psi} - \frac{f(b-b)}{b(b-f)\sin \psi}$$

Hincque denique aſſignari poterit diſtantia perihelii a ſole $a = \frac{bf \cos v}{b-f+f \cos v}$, ſicque orbita cometæ cognoſcetur. Q. E. J.

Coroll. 1.

22. Si ex æquatione ſupra inventa:

$$(b-g)f = (b-f)g \cos \phi - (b-f)g \sin \phi \tang v$$

loco tang v eliminemus b, reperitur, $b = \frac{fg - fg \cos \phi + fg \sin \phi \tang v}{f - g \cos \phi + g \sin \phi \tang v}$. Simili vero modo ex tertia obſervatione erit $b = \frac{fb - fb \cos \psi + fb \sin \psi \tang v}{f - b \cos \psi + b \sin \psi \tang v}$

Quibus inter ſe comparatis erit:

 $g(1-\cos \phi)$

$$\begin{array}{l} g(1-\operatorname{coſ}\varphi)(f-h\operatorname{coſ}\psi)+g(f-h\operatorname{coſ}\psi)\operatorname{ſin}\varphi\operatorname{tang}v \\ -h(1-\operatorname{coſ}\psi)(f-g\operatorname{coſ}\varphi)+gh(1-\operatorname{coſ}\varphi)\operatorname{ſin}\psi\operatorname{tang}v = v. \\ \qquad -h(f-g\operatorname{coſ}\varphi)\operatorname{ſin}\psi\operatorname{tang}v \\ \qquad -gh(1-\operatorname{coſ}\psi)\operatorname{ſin}\varphi\operatorname{tang}v \end{array}$$

ſeu $f(g-h)-g(f-h)\operatorname{coſ}\varphi+h(f-g)\operatorname{coſ}\psi+g(f-h)\operatorname{ſin}\varphi\operatorname{tang}v$ $-h(f-g)\operatorname{ſin}\psi\operatorname{tang}v=o$ unde invenitur

$$\operatorname{tang} v = \frac{f(h-g)-g(h-f)\operatorname{coſ}\varphi+h(g-f)\operatorname{coſ}\psi}{h(g-f)\operatorname{ſin}\Psi-g(h-f)\operatorname{ſin}\varphi},$$

Coroll. 2.

23. Si alter angulus φ æquetur duobus rectis ita ut rectæ F S, F G in directum ſint poſitæ ob ſin $\varphi = o$ & tang $\varphi = o$, fiet $h = \dfrac{\frac{1}{\operatorname{tang}\varphi}-\frac{1}{\operatorname{ſin}\varphi}}{\frac{1}{f\operatorname{tang}\varphi}-\frac{1}{g\operatorname{ſin}\varphi}} = \dfrac{\operatorname{coſ}\varphi-1}{\frac{\operatorname{coſ}\varphi}{f}-\frac{1}{g}}$. At eſt coſ $\varphi = -1$, unde erit $h = \dfrac{2fg}{f+g}$ quæ eſt eximia ſectionum conicarum proprietas jam quidem nota.

Coroll. 3.

24. Expreſſio autem generalis, quam pro valore ſemi-lateris recti invenimus, ſi ad lineas revocetur, ſequens ſuppeditabit theorema, naturam ſectionum conicarum non parum illuſtrans.

Theorema.

Fig. 3. *Si a tribus quibuscunque ſectionis conicæ punctis F. G. H ad ejus alterum focum S ducantur rectæ F S, G S, & H S atque duo eorum G & H cum tertio F jungantur rectis G F & H F, ac denique ex G ducatur recta G I K parallela ipſi F S ſecans rectas H F, H S.*

ſi opus

ſi opus eſt productas in I & K; erit G I ad SK + KG − SG uti S F ad ſemiſſem lateris recti.

Demonſtratio.

25. Vocentur ut ante diſtantiæ $FS = f$; $GS = g$ & $HS = h$ itemque anguli $FSG = \varphi$ & $FSH = \psi$. Demittatur ex S in GIK perpendiculum SL, quod vocetur $= r$. Cum igitur anguli GSL, KSL, ſint complementa angulorum φ & ψ erit $GL = \frac{r}{\text{tang}\,\varphi}$; $KL = \frac{r}{\text{tang}\,\psi}$: $SG = \frac{r}{\text{ſin}\,\varphi}$ & $SK = \frac{r}{\text{ſin}\,\psi}$. Hinc $SK + KG - SG = \frac{r}{\text{ſin}\,\psi} + \frac{r}{\text{tang}\,\varphi} - \frac{r}{\text{tang}\,\psi} - \frac{r}{\text{ſin}\,\varphi}$. Ducatur KM parallela ipſi HF; erit $SH : SK = SF : SM$, ſeu $h : \frac{r}{\text{ſin}\,\psi} = f : SM$; unde fit $SM = \frac{fr}{h\,\text{ſin}\,\psi}$, ideoque $FM = IK = f - \frac{fr}{h\,\text{ſin}\,\psi} = \frac{fr}{r} - \frac{fr}{h\,\text{ſin}\,\psi}$. At ob $SG = \frac{r}{\text{ſin}\,\varphi} = g$ erit $r = g\,\text{ſin}\,\varphi$, hincque $IK = \frac{fr}{g\,\text{ſin}\,\varphi} - \frac{fr}{h\,\text{ſin}\,\psi}$. Quare $GI = GK - KI = \frac{r}{\text{tang}\,\varphi} - \frac{r}{\text{tang}\,\psi} - \frac{fr}{g\,\text{ſin}\,\varphi} + \frac{fr}{h\,\text{ſin}\,\psi}$. Cum igitur invenerimus eſſe

$$h = \frac{\frac{1}{\text{tang}\,\varphi} - \frac{1}{\text{ſin}\,\varphi} - \frac{1}{\text{tang}\,\psi} + \frac{1}{\text{ſin}\,\psi}}{\frac{1}{f\,\text{tang}\,\varphi} - \frac{1}{g\,\text{ſin}\,\varphi} - \frac{1}{f\,\text{tang}\,\psi} + \frac{1}{h\,\text{ſin}\,\psi}} =$$

$$\frac{\frac{r}{\text{tang}\,\varphi} - \frac{r}{\text{ſin}\,\varphi} - \frac{r}{\text{tang}\,\psi} + \frac{r}{\text{ſin}\,\psi}}{\frac{r}{\text{tang}\,\varphi} - \frac{r}{g\,\text{ſin}\,\varphi} - \frac{r}{\text{tang}\,\psi} + \frac{fr}{h\,\text{ſin}\,\psi}}\, f,$$ erit ſemi-latus rectum

$h =$

$b = \frac{SK + KG - SG}{GI} . SF$, ideoque GI: SF $=$ SK$+$KG$-$SG: ſemi-latus rectum. Q. E. D.

27. Hactenus temporum, quæ inter obſervationes effluxerunt nullam habuimus rationem. Nunc igitur cognita orbita ope præceptorum ſupra traditorum definiri poterunt tempora, quibus cometa a perihelio A, ad unumquodque trium punctorum F, G, & H pervenire debuit, hisque inventis prodibunt temporum intervalla inter obſervationes elapſa, quæ ſi cum temporibus obſervatis convenire deprehendantur, hoc certum erit ſignum, poſitionem lineæ nodorum atque inclinationem orbitæ cometæ ad eclipticam recte eſſe aſſumtas. Sin autem hæc tempora per calculum eruta cum obſervatis minus conguant, concludendum erit vel in aſſumtione lineæ nodorum, vel inclinationis vel utriusque eſſe aberratum. Qui error ſi fuerit ſatis parvus, debebit autem eſſe talis, ſi quidem hæc elementa jam per methodum primam proxime noverimus, ex ipſa aberrationis quantitate corrigi poterit. Et quia duo habemus temporum intervalla, quæ cum calculo comparari poſſunt, duæ reſultabunt comparationes, quæ ad utrumque errorem, qui forte ſint commiſſi, emendandum ſufficient. Ita igitur tres cometæ obſervationes ſufficient ad ejus orbitam vere cognoſcendam.

27. Poſtquam ergo ope methodi ante expoſitæ orbita cometæ jam propemodum fuerit determinata, exinde tam poſitio lineæ nodorum quam inclinatio orbitæ ad eclipticam ſatis exacte ad hoc inſtitutum cognoscentur; neque etiam opus erit illum

calculum

calculum ſummo ſtudio ad finem perducere, ſed ſufficiet ope conſtructionum geometricarum hæc momenta ſaltem circiter determinaviſſe; quo pacto totus labor mirifice contrahi poterit. Quando igitur eousque tantum fuerit perventum, ut certo ſciamus lineam nodorum una cum inclinatione ad eclipticam non multum a veritate abhorrere: ſtatim hac altera methodo uti licebit, quam in ſequente problemate explicabo.

Problema III.

Cognitis jam propemodum poſitione lineæ nodorum & inclinatione orbitæ cometæ ad eclipticam, veram cometæ orbitam ex tribus obſervationibus exacte determinare.

Solutio.

28. Sit L longitudo nodi puta aſcendentis per ſuperiorem methodum inventa, & I ſit inclinatio orbitæ ad eclipticam, ibidem eruta. Conſtituantur tres hypotheſes, in quarum prima longitudo nodi aſſumatur $= L$ & inclinatio $= I$. In ſecunda hypotheſi longitudo nodi denuo ſtatuatur $= L$, inclinatio autem aſſumatur $= I + \eta$, ſumendo pro η unum, duosve gradus, prouti errorem minorem majoremve æſtimamus. In tertia hypotheſi ſtatuatur longitudo nodi $= L + \lambda$, inclinatio vero maneat $= I$, ubi iterum ipſi λ valor arbitrarius tribui poteſt, major minorve, prout diſcrepantiam ipſius L a veritate majorem minoremve judicamus. Quo autem propius jam noverimus valores ipſarum L & I, eo minores poterimus litteras η & λ aſſumere, dummodo veritas inter tres hypotheſes tanquam limites contineatur. His hypotheſibus conſtitutis eligantur tres cometæ obſervationes quæcunque, quæ quo longius a ſe invicem fuerint remotæ, eo accuratiorem reddent determinationem: interim tamen evitari debebunt ejusmodi obſervationes, quæ ſint inſtitutæ tum, cum terra prope lineam nodorum verſaretur.

Deinde ad tempora harum obſervationum ſupputentur loca ſolis, cum ejus a terra diſtantiis, tum ſecundum problema primum hujus additamenti pro unaquaque hypotheſi definiantur cometæ a ſole diſtantiæ, atque elongationes a linea nodorum. Quo facto ex ſuperiori problemate determinetur orbita cometæ unicuique hypotheſi conveniens, ſicque obtinebuntur tres diverſæ orbitæ, inter quas vera continebitur; Ad quam inveniendam calculo eruantur temporum intervalla, quæ ſecundum quamlibet hypotheſin inter obſervationes binas præterlabi debuiſſent. Sit T tempus inter primam & ſecundam obſervationem, quod ex prima hypotheſi invenitur; $T + p$ tempus quod ex ſecunda, & $T + q$ tempus quod ex tertia hypotheſi eruitur. Tempus autem obſervatum ſit $= T + k$. Si jam ſumamus in orbita cometæ vera eſſe longitudinem nodi accendentis $= L + x$ & inclinationem ad eclipticam $= I + y$, ut res hoc modo ſe habeat.

	Hypoth. I	Hyp. II	Hyp. III	Orbita vera
Longitudo nodi —	L	L	$L + \lambda$	$L + x$
Inclinatio — —	I	$I + \eta$	I	$I + y$
Tempus inter I & II obſervationem	T	$T + p$	$T + q$	$T + k$

Nunc ita ratiocinemur, quoniam ſi inclinationi I augmentum $= \eta$ tribuitur, tempus T incrementum capit $= p$ incrementum y inclinationis dabit temporis incrementum $= \frac{py}{\eta}$. Deinde quoniam longitudinis nodi incrementum λ producit temporis incrementum $= q$, longitudinis L incrementum $= x$ producet temporis incrementum $= \frac{qx}{\lambda}$. Quare ex orbita vera erit intervallum temporis inter primam & ſecundam obſervationem $= T + \frac{py}{\eta} + \frac{qx}{\lambda}$, quod cum

æquale

æquale eſſe debeat tempori obſervato T + k: habebitur æquatio $\frac{py}{\eta} + \frac{qx}{\lambda} = k$. Similis æquatio elicietur ex intervallo temporis inter primam & tertiam obſervationem, atque ex his duabus æquationibus definientur quantitates x & y, hincque innoteſcent & vera longitudo nodi aſcendentis $= L + x$, & vera inclinatio orbitæ ad eclipticam $= I + y$. Denique cum ex comparatione trium aſſumtarun hypotheſium conſtet, quantum incrementa η & λ cum latus rectum orbitæ b tum diſtantiam perihelii a ſole a, item anomaliam veram primæ obſervationis v una cum tempore a perihelio ad primam obſervationem elapſo, immutaverint, per regulam auream definientur harum rerum mutationes ex incrementis x & y ſimul ſumtis oriundæ. Hocque modo determinabuntur orbitæ veræ latus rectum, diſtantia perihelii a ſole, elongatio perihelii a loco primæ obſervationis, ideoque etiam ab altero, ac tandem tempus quo cometa per perihelium tranſierit. Q. E. J.

Coroll. 1.

29. Cognitis anomaliis veris cometæ in ſingulis obſervationibus, tempora, quibus eæ a tranſitu cometæ per perihelium diſtant modo ſupra expoſito aſſignabuntur. Sit enim a diſtantia perihelii a ſole, b ſemi-latus rectum & v anomalia vera cometæ in obſervatione propoſita. Ponatur $\frac{2a-b}{b} = n$ erit n numerus valde parvus ſi orbita proxime fuerit parabolica, ſitque $t = \text{tang } \frac{1}{2} v$. & quæratur valor ſequentis ſeriei

$$S = t + \tfrac{1}{3}t^3 - \tfrac{2}{5}nt^5 + \tfrac{3}{7}n^2t^7 - \tfrac{4}{9}n^3t^9 + \tfrac{5}{11}n^4t^{11} \\ + \tfrac{3}{5}n^2t^5 - \tfrac{4}{7}n^3t^7 + \tfrac{5}{9}n^4t^9 - \tfrac{6}{11}n^5t^{11} \ \&c.$$

quo invento erit tempus quæſitum in diebus expreſſum $= \frac{aa}{m\sqrt{b}} S$, exiſtente $m = 271989, 739$ &c. $lm = 5, 4345525139$.

Coroll. 2.

30. Si correctiones x & y, quæ hoc modo inveniuntur prodeant nimis magnæ, minus quoque erunt accuratæ. Quod enim poſuimus

mus variationes ab incrementis, quas longitudini nodi & inclinationi tribuimus, oriundas his ipſis incrementis eſſe proportionales, hoc tantum locum habere poteſt, ſi incrementa fuerint minima. Interim tamen iſte labor non erit inutilis, ſed ejus ope cognoſcemus poſitionem lineæ nodorum & inclinationem orbitæ ad eclipticam multo propius, quam aſſumſeramus; ſicque exinde novas hypotheſes veritati magis conſentaneas & inter ſe propiores formare poterimus. Quo facto ſi idem calculus, qui in ſolutione hujus problematis eſt præſcriptus, denuo inſtituatur, tum exacte orbita cometæ vera cognoſcetur, quantum quidem per minimos errores, quibus etiam exactiſſimæ obſervationes ſunt obnoxiæ, ſperare licet.

31. Hac igitur methodo uſus ſum ad orbitam cometæ priori methodo repertam corrigendam. Quoniam vero obſervationes, quæ tum erant in poteſtate, nimis longe a perihelio erant remotæ, inclinatio imprimis ad eclipticam nimis prodiit incerta, quoniam parva mutatio in hypotheſi diſtantiæ cometæ a terra facta differentiam aliquot graduum produxerat. Hanc obrem coactus fui calculum hic traditum bis applicare: atque ex prima operatione ſtatim cognovi inclinationem orbitæ ad eclipticam notabiliter minorem eſſe, quam ſupra eſſem ſuſpicatus, atque 47° vix excedere. Feci igitur pro ſecunda operatione hypotheſes cum inter ſe tum veritati propiores. Elegi ad hoc inſtitutum ex obſervationibus a Celeb. Caſſino mecum communicatis primam & ultimam, utpote maxime a ſe invicem remotas, ac propterea orbitam cometæ accuratiſſime determinantes; cum ultima fere in ipſo perihelio ſit facta, ubi motus erat velociſſimus, ac minima aberratio ſatis ſenſibile diſcrimen producere debebat. Præterea vero elegi obſervationem penultimam, quæ etſi ratione temporis ultimæ nimis vicina videatur, tamen angulus, quem cometa interea circa ſolem deſcripſit, ſatis eſt notabilis, ita ut iſta tria puncta aptiſſima videantur ad orbitam determinandam. Hunc igitur calculum integrum, quo ſecunda operatio conſtabat, hic apponam, prætermiſſo priori, quippe cujus ratio ex hoc facile perſpicitur.

Tempo-

Tempore medio Pariſino St. novi.

A. 1743.	Long. Cometæ	Lat. Com. boreal.	
Dec. 21[d], 6[h], 57′	0[s],22°,23′,0″	16°,18′,57″ = *p*	
Locus Solis.	8, 29, 36, 0	4, 992675 = *l*S T	
t = S T N =	3, 22, 47, 0 =	112°,47′,0″	
	Hypoth. I.	Hypoth. II.	Hypoth. III
Incl. ad Ecl. = *i* =	46°,50′,0″	47°,10′,0″	46°,50′,0″
Long. ☊ helioc. =	1[s],15, 50, 0	1, 15,50,0	1, 16,10,0
Long. Terræ =	2, 29, 36, 0	2, 29,36,0	2, 29,36,0
s = TSN =	43, 46, 0	43, 46, 0	43, 26, 0
t = STN =	112, 47, 0	112, 47, 0	112, 47, 0
s + *t* =	156, 33, 0	156, 33, 0	156, 13, 0
n = SNT =	23, 27, 0	23, 27, 0	23, 47, 0
A *l* coſ *i* =	9, 835134	9, 832425	9, 835134
ſubtr. { *l* tang *s* =	9, 981297	9, 981297	9, 976238
{ *l* ſin *s* =	9, 839932	9, 839932	9, 837279
l coſ *i*: tang *s* =	9, 853837	9, 851128	9, 858896
l coſ *i*: ſin *s* =	9, 995202	9, 992493	9, 997855
Ad *l* tang *p* =	9, 466453	9, 466453	9, 466453
add. *l* ſin *s* =	9, 839932	9, 839932	9, 837279
	9, 306385	9, 306385	9, 303732
ſubtr. a *l* ſin *i* =	9, 862946	9, 865302	9, 862946
	0, 556561	0, 558917	0, 559214
add { *l* ſin *t* =	9, 964719	9, 964719	9, 964719
{ *l* ſin *n* =	9, 599827	9, 599827	9, 605606
l ſin *t*. ſin *i*: ſin *s*. tang *p* =	0, 521280	0, 523636	0, 523933
l ſin *n*. ſin *i*: ſin *s*. tang *p* =	0, 156388	0, 158744	0, 164820
coſ *i*: tang *s* =	0, 714228	0, 709787	0, 722597
ſin *t*. ſin *i*: ſin *s*. tang *p* =	3, 321080	3, 339160	3, 341440
cot. CSN =	4, 035308	4, 048947	4, 064037
l cot. CSN =	10, 605876	10, 607342	10, 608958

Ergo CSN =	13°,55′,5″	13°,52′,25″	13°,49′,25′
coſ *i*: ſin *s* =	0, 989015	0, 982865	0, 995075
ſin *n*. ſin *i*: ſin *s*. tang *p* =	1, 433470	1, 441270	1, 461576
	2, 422485	2, 424135	2, 456651
log: =	0, 384260	0, 384556	0, 390343
a *l* ST =	4, 992675	4, 992675	4, 992675
l CP =	4, 608415	4, 608119	4, 602332
ſubtr. *l* ſin CSN =	9, 381170	9, 379810	9, 378270
l SC =	5, 227245	5, 228309	5, 224062
SC =	168754	169165	167518

Tempore medio Pariſino St. novi

A. 1744.	Long. Com.	Lat. Com. Bor.
Febr. 25^d, 5^h, 36′	11^{s}9°,52′,46″	14°,39′,7″ = *p*
Locus ſolis	11, 6, 31, 37	4,996003 = *l* ST
t = STN =	3, 21, 10	

	Hypoth. I	Hypoth. II	Hypoth. III
Incl. ad Ecl. = *i* =	46, 50, 0	47, 10, 0	46, 50, 0
Long. ☊ helioc.	1, 15,50,0	1, 15,50,0	1, 16,10,0
Long. Terræ −	5, 6,31,40	5, 6,31,40	5, 6,31,40
s = TSN =	3, 20,41,40	3, 20,41,40	3, 20,21,40
t = STN =	3,21,10	3,21,10	3,21,10
s + *t* =	3, 24, 2,50	3, 24, 2,50	3, 23,42,50
n = SNT =	65,57,10	65,57,10	66,17,10
A *l* coſ *i* =	9, 835134	9, 832425	9, 835134
ſubtr. { *l* − tang *s* =	10, 422787	10, 422787	10, 430481
ſubtr. { *l* ſin *s* =	9, 971034	9, 971034	9, 971980
l − coſ *i*: tang *s* =	9, 412347	9, 409638	9, 404653
l coſ *i*: ſin *s* =	9, 864100	9, 861391	9, 863154
Ad *l* tang *p* =	9, 417386	9, 417386	9, 417386
add *l* ſin *s* =	9, 971034	9, 971034	9, 971980

a *l* ſin *i* =

	9, 388420	9, 388420	9, 389366
a *l* ſin *i* =	9, 862946	9, 865302	9, 862946
	0, 474526	0, 476882	0, 473580
add. { *l* ſin *t* =	8, 767038	8, 767038	8, 767038
add. { *l* ſin *n* =	9, 960571	9, 960571	9, 961689
l ſin *t*. ſin *i*: ſin *s*. tang *p* =	9, 241564	9, 243920	9, 240618
l ſin *n*. ſin *i*: ſin *s*. tang *p* =	0, 435097	0, 437453	0, 435269
— coſ *i*: tang *s* =	0, 258433	0, 256826	0, 253895
ſin *t*. ſin *i*: ſin *s*. tang *p* =	0, 174407	0, 175356	0, 174027
— cot. CSN =	0, 084026	0, 081470	0, 079868
l — cot. CSN =	8, 924414	8, 910998	8, 902373
Ergo CS☋ =	85, 11,45	85, 20,35	85, 26,0
coſ *i*: ſin *s* =	0, 731308	0, 726760	0, 729716
ſin *n*. ſin *i*: ſin *s*. tang *p* =	2, 723313	2, 738135	2, 724390
	3, 454621	3, 464895	3, 454106
log =	0, 538400	0, 539690	0, 538335
a *l* ST =	4, 996003	4, 996003	4, 996003
l CP =	4, 458603	4, 456313	4, 457668
ſubtr. *l* ſin CS☋ =	9, 998471	9, 998564	9, 998619
l SC =	4, 460132	4, 457749	4, 459049
S C =	28849	28691	28777

Tempore medio Pariſino St. novi

A. 1744	Long. Cometæ	Lat. Com. bor.	
Febr. 29ᵈ, 18ʰ, 57′	11ˢ,2°,32′,0″	6°,28′,21″	= *p*
Locus ſolis =	11, 11, 5,40	4, 996506	= *l* ST
— *t* = S T N =	8°,33′,40″		

	Hypoth. I	Hypoth. II	Hypoth. III
Incl. ad Ecl. = *i* =	46°,50′,0″	47°,10′,0″	46°,50′,0″
Long. ☊ helioc. =	1ˢ,15,50,0	1ˢ,15,50,0	1ˢ,16,10,0
Long Terræ =	5, 11, 5,40	5, 11, 5,40	5, 11, 5,40
s = TSN =	3, 25,15,40	3, 25,15,40	3, 24,55,40
— *t* = STN =	8,33,40	8,33,48	8,33,40

s + *t* =

$s+t$ =	3, 16,42,0	3, 16,42,0	3, 16,22,0
n = SNT =	73,18,0	73,18,0	73,39,0
A l coſ i =	9, 835134	9, 832425	9, 835134
ſubtr. { l tang s =	10, 326180	10, 326180	10, 332758
ſubtr. { l ſin s =	9, 956347	9, 956347	9, 957531
l — coſ i: tang s =	9, 508954	9, 506245	9, 502376
l coſ i: ſin s =	9, 878787	9, 876078	9, 877603
Ad l tang p =	9, 054784	9, 054784	9, 054784
add. l ſin s =	9, 956347	9, 956347	9, 957531
	9, 011131	9, 011131	9, 012315
a l ſin s =	9, 862946	9, 865302	9, 862946
	0, 851815	0, 854171	0, 850631
add. { l—ſin t =	9, 172790	9, 172790	9, 172790
add. { l ſin n =	9, 981285	9, 981285	9, 982035
l—ſin t.ſin i: ſin s: tang p =	0, 024605	0, 026961	0, 023421
l ſin n. ſin i: ſin s: tang p =	0, 833100	0, 835456	0, 832666
— coſ i: tang s =	0, 322815	0, 320808	0, 317963
— ſin t. ſin i: ſin s. tang p =	1, 058290	1, 064050	1, 055410
— cot CSN =	1, 381105	1, 384858	1, 373373
l — cot. CSN =	10, 140226	10, 141405	10, 137788
Ergo CS☊ =	35,54,25	35,50,0	36, 3,35
coſ i: ſin s =	0, 756462	0, 751759	0, 754404
ſin n. ſin i: ſin s. tang p =	6, 809265	6, 846300	6, 802460
	7, 565727	7, 598059	7, 556864
log: =	0, 878850	0, 880702	0, 878341
a l ST =	4, 996506	4, 996506	4, 996506
l CP =	4, 117656	4, 115804	4, 11816[illegible]
ſubtr. l ſin CS☋ =	9, 768246	9, 767474	9, 769840
l SC =	4, 349410	4, 348330	4, 348325
SC =	22357	22301½	22301

A. 1744. Febr.	Hypoth. I	Hypoth. II	Hypoth. III
$29^d, 18^h, 57' f =$	22357	$22301\frac{1}{2}$	22301
$lSF = lf =$	4, 349410	4, 348330	4, 348325
☋ SF =	35,54,25	35,50,0	36, 3, 25
A. 1744. Febr. $25^d, 5^h, 36' g =$	28849	28691	28777
$lSG = lg =$	4, 460132	4, 457749	4, 459049
☋ SG =	85, 11,45	85 20,35	85,26, 0
A. 1743. Dec. $21^d, 6^d, 57' h =$	168754	169165	167518
$lSH = lh =$	5, 227245	5, 228309	5, 224062
☋ SH =	166, 4, 55	166, 7, 35	166,10,35
$FSG = \phi =$	49, 17, 20	49, 30, 35	49, 22, 25
$FSH = \psi =$	130, 10, 30	130, 17, 35	130,7 , 0
$180 - \psi = \chi =$	49, 49, 30	49, 42, 25	49, 53, 0
$l \frac{1}{\text{tang}\,\chi} = l \cot \chi =$	9, 926506	9, 928321	9, 925609
$l f =$	4, 349410	4, 348330	4, 348325
$l \frac{1}{f \text{tang}\,\chi} =$	5, 577096	5, 579991	5, 577284
$l \sin \chi =$	9, 883137	9, 882380	9, 883510
$l \frac{1}{\sin \chi} =$	0, 116862	0, 117619	0, 116489
$l b =$	5, 227245	5, 228309	5, 224062
$l \frac{1}{b \sin \chi} =$	4, 889617	4, 889310	4, 892427
$l \frac{1}{\text{tang}\,\phi} =$	9, 934737	9, 931342	9, 933437
$l f =$	4, 349410	4, 348330	4, 348325
$l \frac{1}{f \text{tang}\,\phi} =$	5, 585327	5, 583012	5, 585112

	Hypoth. I	Hypoth. II	Hypoth. III
l ſin Φ =	9, 879674	9, 881109	9, 88022[illegible]
$l \frac{1}{\text{ſin}\,\Phi}$ =	0, 120325	0, 118890	0, 119774
$l g$ =	4, 460132	4, 457749	4, 459049
$l \frac{1}{g\,\text{ſin}\,\Phi}$ =	5, 660193	5, 661141	5, 660725
1: tang χ =	0, 844319	0, 847854	0, 842576
1: ſin χ =	1, 308770	1, 311055	1, 307643
1: tang Φ =	0, 860472	0, 853772	0, 857900
	3, 013561	3, 012681	3; 008119
1: ſin Φ =	1, 319246	1, 314892	1, 317573
Numer:	1, 694315	1, 697789	1, 690546
l Num:	0, 228994	0, 229883	0, 228026
1: f tang χ =	377656	380182	377820
1: f tang Φ =	384882	382835	384691
1: b ſin χ =	77556	77502	78060
	840094	840519	840571
1: g ſin Φ =	457292	458291	457852
Denom: =	382802	382228	382719
l Denom: =	5, 582974	5, 582322	5, 582879
l Num: =	0, 228994	0, 229883	0, 228026
$l\,b$ =	4, 646020	4, 647561	4, 645147
b =	44261	44418	44172
g =	28849	28691	28777
f =	22357	22301	23301
$b - g$ =	15412	15727	15395
$b - f$ =	21904	22117	21871
$l f$ =	4, 349410	4, 348330	4, 348325
l 1: g ſin Φ =	5, 660193	5, 661141	5, 660725
$l\,b - g$ =	4, 187159	4, 196646	4, 187380

$l\,(b-f)$ =

	Hypoth. I	Hypoth II	Hypoth III
	4, 196762	4, 206117	4, 196430
l (*b*—*f*) =	4, 340523	4, 344726	4, 339869
	9, 856239	9, 861391	9, 856561
Numer. =	0, 718190	0, 726760	0, 718723
a cot φ =	0, 860472	0, 853772	0, 857900
tang *v* =	0, 142282	0, 127012	0, 139177
l tang *v* =	9, 153150	9, 103844	9, 143566
v =	8°, 5′,50″	7°,14′,10″	7°,55′,20″
l coſ *v* =	9, 995648	9, 996527	9, 995835
l f =	4, 349410	4, 348330	4, 348325
l f coſ *v* =	4, 345058	4, 344857	4, 344160
l b =	4, 646020	4, 647561	4, 645147
l Num: =	8, 991078	8, 992418	8, 989307
f coſ *v* =	22134	22123	22088
b —*f* =	21904	22117	21871
Denom: =	44038	44240	43959
l Denom: =	4, 643827	4, 645815	4, 643048
A *l* Num: =	8, 991078	8, 992418	8, 989307
l a =	4, 347251	4, 346603	4, 346259
a =	22246	22213	22195
2 *a* =	44492	44426	44390
b =	44261	44418	44172
2 *a* — *b* =	231	8	218
l (2 *a* — *b*) =	2, 363612	0, 903090	2, 338456
l b =	4, 646020	4, 647561	4, 645147
l n =	7, 717592	6, 255529	7, 693309
l aa =	8, 694502	8, 693206	8, 692518
l √ *b* =	2, 323010	2, 323780	2, 322573
	6, 371492	6, 369426	6, 369945
l m =	5, 434553	5, 434553	5, 434553

 l N =

l N $=$	0, 936939	0, 934873	0, 935392
$\frac{1}{2}v$ $=$	4°,2′,55″	3, 37, 5	3, 57, 40
$l\,t$ $=$	8, 849906	8, 800928	8, 840387
$l\,t^3$ $=$	6, 549718	6, 402784	6, 521161
$l\,3$ $=$	0, 477121	0, 477121	0, 477121
$l\,\frac{1}{3}t^3$ $=$	6, 072597	5, 925663	6, 044040
l N $=$	0, 936939	0, 934873	0, 935392
l N t $=$	9, 786845	9, 735801	9, 775779
$l\,\frac{1}{3}$ N t^3 $=$	7, 009536	6, 860536	6, 979432
N t $=$	0, 61213	0, 54425	0, 59673
$\frac{1}{3}$ N t^3 $=$	0, 00102	0, 00072	0, 00095
A Perih. ad I.	0, 61315	0, 54497	0, 59768
obſervationem	14^h,42′	13^h,5	14^h,20′
Com. per Perih.	29, 18, 57	29, 18,57	29, 18,57
trans: 1744. Mart.	1^d,9^h,39′	1^d,8^h,2′	1^d,9^h,17′
$\frac{1}{2}v$ $=$	4, 2, 55	3, 37, 5	3, 57, 40
$\frac{1}{2}\phi$ $=$	24, 38, 40	24, 45, 17½	24, 41, 12⅓
$\frac{1}{2}v^1$ $=$	28, 41, 35	28, 22, 20	28, 38, 50
$l\,t$ $=$	9, 738250	9, 732451	9, 737421
$l\,t^3$ $=$	9, 214750	9, 197353	9, 212263
$l\,3$ $=$	0, 477121	0, 477121	0, 477121
$l\,\frac{1}{3}t^3$ $=$	8, 737629	8, 720232	8, 735142
$l\,t^5$ $=$	8, 691250	8, 662255	8, 687105
$l\,n$ $=$	7, 717592	6, 255529	7, 693309
$l\,n\,t^5$ $=$	6, 408842	5, 977784	6, 380414
$l\,n^2\,t^5$ $=$	4, 126434	2, 173313	4, 073723
t $=$	0, 54733	0, 54007	0, 54629
$\frac{1}{3}t^3$ $=$	0, 05465	0, 05251	0, 05434
	0, 60198	0, 59258	0, 60063
ſubtr. $\frac{2}{5}n\,t^2$ $=$	0, 00010	0, 00003	0, 00010

S $=$

	Hypoth. I.	Hypoth. II.	Hypoth. III
$S =$	0, 60188	0, 59255	0, 60053
$lS =$	9, 779510	9, 772725	9, 778535
$lN =$	0, 936939	0, 934873	0, 935392
A. Perih: ad	0, 716449	0, 707598	0, 713927
II Obſerv: dier:	5, 2053	5, 1003	5, 1752
A Per. ad I - -	0, 6131	0, 5449	0, 5977
A I ad II - - - -	4, 5922	4, 5554	4, 5775
$\frac{1}{2} v =$	4, 2, 55	3, 37, 5″	3, 57, 40
$\frac{1}{2} \psi =$	65, 5, 15	65, 8, 37	65, 3, 30
$\frac{1}{2} v'' =$	69, 8, 10	69, 45, 40	69, 1, 10
$lt =$	0, 418914	0, 410435	0, 416263
$lt^3 =$	1, 256742	1, 231305	1, 248789
$l3 =$	0, 477121	0, 477121	0, 477121
$l\frac{1}{3}t^3 =$	0, 779621	0, 754184	0, 771668
$lt^5 =$	2, 094570	2, 052175	2, 081315
$ln =$	7, 717592	6, 255529	7, 693309
$lnt^5 =$	9, 812162	8, 307704	9, 774624
$ln^2t^5 =$	7, 529754	4, 563233	7, 467933
$lt^2 =$	0, 837828	0, 820870	0, 832526
$ln^2t^7 =$	8, 367582	5, 384103	8, 300459
$ln =$	7, 717592	6, 255529	7, 693309
$ln^3t^7 =$	6, 085174	1, 639632	5, 993768
$lt^2 =$	0, 837828	0, 820870	0, 832526
$ln^3t^9 =$	6, 923002	2, 460502	6, 826294
$ln =$	7, 717592	- - - -	7, 693309
$ln^4t^9 =$	4, 640594		4, 519603
$ln^4t^{11} =$	5, 478422		5, 352129
$t =$	2, 62370	2, 57297	2, 60774
$+\frac{1}{3}t^3$	6, 02034	5, 67785	5, 91110

$-\frac{2}{3}n^2t^5 =$

	8, 64404	8, 25082	8, 51884
$-\frac{2}{5} n t^5 =$	0, 25956	0, 00812	0, 23806
	8, 38448	8, 24270	8, 28078
$+\frac{2}{5} n^2 t^5 =$	203		176
	8, 38651	8, 24270	8, 28254
$+\frac{3}{7} n^2 t^7 =$	1000	1	856
	8, 39651	0, 24271	8, 29110
$-\frac{4}{7} n^3 t^7 =$	0, 00007		0, 00005
	8, 39644	8, 24271	8, 29105
$-\frac{4}{9} n^3 t^9 =$	0, 00037		0, 00029
	8, 39607	8, 24271	8, 29076
$+\frac{5}{11} n^4 t^{11} =$	1		1
S =	8, 39608	8, 24271	8, 29077
l S =	0, 924076	0, 916070	0, 918595
l N =	0, 936939	0, 934873	0, 935392
A Perihelio	1, 861015	1, 850943	1, 853987
a III Obſerv. dies	72, 6131	70, 9485	71, 4475
A Perih. ad I	0, 6131	5, 5449	0, 5977
A I ad III	72, 0000	70, 4036	70, 8498

32. Si igitur correctionem in §. 28. traditam adhibere velimus, fiet: L = 1, 15, 50, 0 & I = 46°, 50′, 0″ item λ = 20′ & η = 20′. Pōnatur ergo pro orbita vera Longitudo nodi aſc. ☊ = 1ˢ, 15°, 50′ + x'
Inclinatio orbitæ ad Eclipt. = 46°, 50′ + y'
Conſideremus nunc intervallum temporis inter primam & ſecundam obſervationem, quod revera erat. 4^d, 13^h, 21′ = 4, 5562 dies. Calculo vero ita prodiit

Tempus a I ad II	Hypoth. I	Hypoth. II	Hypoth. III
	4, 5922	4, 5554	4, 5775
T + k =	4, 5562	4, 5922	4, 5922

k = − 360; p = − 368; q = − 147

Cum

Cum ergo ſit $\frac{py}{\eta} + \frac{qx}{\lambda} = k$ erit $360 = 18, 4\,y + 7,3\,x$

Simili modo cum tempus inter primam & tertiam obſervationem fuerit $= 70^d, 12^h = 7, 5000$, hypotheſes vero habeant.

Tempus ad I ad III	Hyp. I	Hyp. II	Hyp. III
	72, 0000	70, 4036	70, 8498
$T + k =$	70, 5000	72, 0000	72, 0000

$k = -1, 5000\ p = -1, 5964\ q = -1, 1502$

Erit ergo $15000 = 798, 2y + 575, 1x$. Ex prima æquatione fit $x = 49, 3 - 2, 5\,y$, qui valor in altera ſubſtitutus dat $6395\,y = 133524$, unde fit: $y = 20', 53''$ hincque erit $x = -3', 54''$. Quare habebitur Pro orbita cometæ vera:

Longitudo nodi aſcendentis - - - $1^s, 15^\circ, 46', 6''$
Inclinatio Orbitæ ad Eclipticam $47^\circ, 10', 53''$

33. Hinc reliqua elementa per interpolationem determinabuntur.

	Hypoth. I	Hypoth. II	Hypoth. III
$=$	22246	22213	22195
		22246	22246

$p = -33\quad q = -51$

Cum igitur ſit quantitas ad a addenda $k = \frac{py}{\eta} + \frac{qx}{\lambda}$ fiet $k = -34 + 10 = -24$, ideoque vere $a = 22222$.

Deinde

Deinde in erat $b =$	Hypoth. I	Hyp. II	Hyp. III
	44261	44418	44172
		44261	44261

$p = 157$; $q = -89$

Ad b ergo addi debet quantitas $k = \frac{py}{\eta} + \frac{qx}{\lambda}$; hoc eſt $k = 164 + 17 = 181$, unde fit vere $b = 44442$

Cometa per Perih. tranſiit A. 1744 Mart.	Hyp. I	Hyp. II.	Hyp. III
	$1^d, 9^h, 39'$	$1^d, 8^h, 2'$	$1^d, 9^h, 17'$
		1, 9, 39	1, 9, 39

$p = -1, 37$ $q = -22'$

Hinc $k = \frac{py}{\eta} + \frac{qx}{\lambda} = -1^h, 38'$. Quare revera:

Cometa per perihelium tranſiit:

A. 1744 Martii $1^d, 8^h, 2'$.

Denique	Hyp. I	Hyp. II	Hyp. III
I. Obſ. an. vera	8°, 5′, 50″	7, 14, 10	7, 55, 20
I. Obſ. a ☋	35, 54, 25	35, 50, 0	36, 3, 35
☋ a Perihel.	27, 48, 35	28, 35, 50	28, 8, 15
		27, 48, 35	37, 48, 35

$p = 47, 15$; $q = 19, 40$

Ergo $k = \frac{py}{\eta} + \frac{qx}{\lambda} = 45', 33''$. Quocirca diſtantia nodi descendentis a perihelio erit 28°, 34′, 8″. Orbita igitur cometæ vera ſequentibus ſex elementis definietur.

I. Diſtantia

I. Diſtantia perihelii a ſole	a	=	22222
	$l\,a$	=	4, 346783
II. Semi-latus rectum . . .	b	=	44442
	$l\,b$	=	4, 647793

III. Cometa per Perihelium tranſiit A. 1744 Mart. $1^d, 8^h, 2'$ tempore medio Pariſino.

IV. Elongatio nodi deſcendentis ☋ a perihelio ſeu angulus AS☋ = $28°, 34', 8''$

V. Longitudo nodi aſcendentis ☊ . $1^s, 15°, 46', 6''$

VI. Inclinatio orbitæ ad Eclipticam $47°, 10', 53''$

34. Orbita ergo hujus cometæ tam parum a parabola diſcrepat, ut ſine ſenſibili errore in calculo pro parabola haberi poſſit. Hinc neceſſe eſt, ut ejus tempus periodicum ſit longiſſimum, plurimisque demum ſeculis abſolvatur; quod quoque experientia confirmatur, cum nullum in tabula Cometarum Hallejana veſtigium inveniamus cujusquam cometæ, cujus orbita ſaltem ratione quorumdam elementorum cum præſenti conveniat. Quoniam autem hæc determinatio cum tribus electis obſervationibus perfecte congruit, dubitare non poterimus, quin eadem & reliquis obſervationibus ſit ſatis factura. Quod ut appareat, ſupputemus ex hac theoria locum cometæ geocentricum pro Februarii $3^d, 8^h, 3\frac{1}{2}'$ tempore medio Pariſino, quo tempore obſervata eſt

longitudo Cometæ $0^s, 0°, 18', 26''$

Latitudo Cometæ $19, 42, 53''$

Ex tabulis autem ſolaribus reperitur pro hoc tempore

Longitudo ſolis	$10^s, 14, 26, 13$
& log. diſt Solis a Terra	4, 993967
Subtrahatur ergo tempus propoſitum Febr.	$3^d, 8^h, 3\frac{1}{2}'$
a tempore perihelii — — — — Febr.	30, 8, 2
erit temp. intervallum =	$27^d - 1\frac{1}{2}'$
ſeu — — — =	26,9989 dieb.

Jam ad anomaliam veram inveniendam, quæratur numerus $N = \frac{aa}{m\sqrt{b}}$.

$la^2 =$	8, 693586
$l\sqrt{b} =$	2, 323896
	6, 369690
$lm =$	5, 434553
$lN =$	0, 935137
ſubtr. a l int. temp.	1, 431346
$l(t + \frac{1}{3}t^3 - \frac{2}{5}nt^5) =$	0, 496209

35. Quoniam vero numerus n tantopere eſt exiguus ut pro nihilo haberi queat, hinc anomalia vera v ex tabula pro motu cometarum parabolico ad finem hujus opuſculi annexa inveniri poterit. Prodibit autem interpolatione adhibita.

$v =$	$117°, 27', 24''$
Perih. a ☊ —	151, 25, 52
Elongatio Cometæ a ☊ —	$33°, 58', 28''$

Ex ano-

Ex anomalia vera v reperitur diſtantia cometæ a ſole SC $= f$. Cum enim ſit $f = \frac{ab}{a + (b-a)\,\text{coſ}\,v}$ ob $b = 2a$ erit $f = \frac{2a}{1 + \text{coſ}\,v} = \frac{a}{(\text{coſ}\,\frac{1}{2}v)^2}$. Cum ergo ſit

$\frac{1}{2}v$	=	58°,43′,42
erit $l\,\text{coſ}\,\frac{1}{2}v$	=	9, 715250
$2\,l\,\text{coſ}\,\frac{1}{2}v$	=	9, 430500
a la	=	4, 346783
Erit lf	=	4, 916283

Hæc igitur ad figuram primam referendo. Cum ſit:

Locus Nodi ☊	—	1ˢ,15°,46′,6″
Long. Terræ	—	4, 14, 26, 13
Erit angulus TSN $= s =$		88°,40,7″

36. Ex determinationibus, quas ſupra in explicatione hujus figuræ invenimus, dum poſuimus: ST $= c$, SC $= f$; CSN $= \Phi$, TSN $= s$, inclinationem orbitæ ad Eclipticam $= i$; atque ang. SNT $= n$, & latitudinem cometæ geocentricam $= p$; ex ſupra inquam inventis formulis colligitur fore: $\text{tang}\,n = \frac{c\,\text{ſin}\,s - f\,\text{ſin}\,\Phi\,\text{coſ}\,i}{f\,\text{coſ}\,\Phi - \text{coſ}\,s}$, &

$$\frac{\text{coſ}\,n}{\text{tang}\,p} = \frac{f\,\text{coſ}\,\Phi - c\,\text{coſ}\,s}{f\,\text{ſin}\,\Phi\,\text{ſin}\,i}.$$

Quæ formulæ fortaſſe commodiores ſunt ad locum geocentricum ex heliocentrico inveniendum, quam modus trigonometricus uſitatus. Cum igitur ſit

$l\,c =$	4, 993967
$l\,f =$	4, 916283
$i =$	47°,10′,53″
$s =$	88°,40, 7″
$\phi =$	33°,58′,28″

calculus ſequenti modo commodiſſime inſtituetur.

	$l\,c =$	4, 993967
add	l ſin s =	9, 999882
	l coſ s =	8, 366080
	$l\,c$ ſin s =	4, 993849
	$l\,c$ coſ s =	3, 359967
	$l\,f =$	4, 916283
add.	l ſin ϕ =	9, 747270
	l coſ ϕ =	9, 918705
	$l\,f$ coſ ϕ =	4, 834988
	$l\,f$ ſin ϕ =	4, 663553
add.	l coſ i =	9, 832300
	l ſin i =	9, 865400
	$l\,f$ ſin ϕ coſ i =	4, 495853
	$l\,f$ ſin ϕ ſin i =	4, 528953
	c ſin s =	98594
	f ſin ϕ coſ i =	31322
	Num: =	67272
	f coſ ϕ =	68389
	c coſ s =	2291

Denom.

Denom. =	66098
l Num : =	4, 827834
l Den. =	4, 820188
l tang n =	10, 007646
Ergo n =	45, 30, 15
add. s =	88, 40, 7
	134, 10, 22
STN = t =	45, 49, 38
ſeu t =	1ˢ, 15, 49, 38
Locus ſolis =	10, 14, 26, 13
Longitudo Cometæ =	0ˢ, 0, 15, 51
l coſ n =	9, 845630
$l. f$ ſin ϕ ſin i =	4, 528953
	4, 374583
l Denom. =	4, 820188
l tang p =	9, 554395
Latitudo Geocentr.	19ᵒ, 43′, 5″

Longitudo ergo per calculum inventa 2′, 35″, & latitudo tantum 12″ ab obſervationibus diſcrepant. Hujus modi autem tantilli errores cum adeo in planetis majores quandoque occurrere ſoleant, merito condonantur.

37. Determinemus jam temporis momenta, quibus cometa per ſuos nodos tranſierit. Quorum locorum cum conſtent anomaliæ veræ, calculus ita ſe habebit.

	Pro ☊	Pro ☋	
Anomalia vera v =	151, 25,52	28, 34, 8	
$\frac{1}{2} v$ =	75, 42,55	14, 17, 5	
$l\,t$ =	0, 594110	9, 405870	
$l\,t^3$ =	1, 782330	8, 217610	
$l\,3$ =	0, 477121	0, 477121	
$l\frac{1}{3}t^3$ =	1, 305209	7, 740489	
t =	3, 9274	0, 25461	
$\frac{1}{3}t^3$ =	20, 1934	0, 00550	
S =	24, 1208	0, 26011	
lS =	1, 382392	9, 415157	
l N =	0, 935137	0, 935137	
	2, 313529	0, 350294	
Temp. a Perihelio	207, 744	2, 2402	
ſeu in diebus:	$207^d, 17^h, 50$	$2^d, 5^h, 45'$	
Perihelium accidit A.1743.Aug.	214, 8′, 2	1, 8, 2	Mart.1744
A. 1743. Aug.	6, 14, 12	3, 13, 47	Mart. 1744

Hinc cognoſcimus cometam per ſuum nodum aſcendentem jam tranſiiſſe anno 1743 Menſis Auguſti die 7mo mane: Per nodum autem deſcendentem tranſiit A. 1744 Mart. d. 3, 13^h, $47'$ tempore Pariſino medio, quæ ſatis congruunt cum iis, quæ ſupra jam ex orbita cometæ minus exacte cognita derivavimus. Ex obſervationibus autem Caſſinianis quilibet facile concludet, cometam hoc circiter tempore eclipticam trajicere debuiſſe, propterea quod ejus latitudo a Febr. d. 25. usque ad 29^{mum} jam octo gradibus decreviſſet, hocque poſteriori tempore adhuc tantum eſſet

6 graduum

6 graduum, quos æſtimatione circiter tribus diebus conficere debuiſſet.

38. Omnino ergo citra eclipticam iſte cometa commoratus eſt 209^d, 23^h, $35'$, quod tempus ſi a toto tempore periodico, quod plurium eſt ſeculorum ſubtrahatur, relinquetur ſpatium temporis, quo cometa in hemisphærio au- rali eſt verſatus; ſicque ſumma inæqualitas inter ejus moram cis & trans eclipticam perſpicitur. Cum autem ab Auguſto menſe anni elapſi in regiones boreales migraſſet, tamdiu ab ecliptica boream verſus receſſit, quoad ejus latitudo heliocentrica maxima ſeu ipſi inclinationi orbitæ æqualis eſſet facta, quod evenit dum ab utroque nodo angulo recto eſſet elongatus. Simili modo poſt tertium diem Martii auſtrum verſus ab ecliptica receſſit, donec a nodo deſcendente eſſet angulo 90° remotus. Quando ergo iſtæ binæ cometæ ab ecliptica maximæ elongationes evenerint investigemus.

	Pro maxima Elongatione boreali	Auſtrali
Anomalia vera Cometæ v =	61°,25′52′	118°,34′,8″
$\frac{1}{2}v$ =	30°,42,56	59, 17′,4
$l\,t$ =	9, 773875	0, 226123
$l\,t^3$ =	9, 321625	0, 678369
$l\,3$ =	0, 477121	0, 477121
$l\,\frac{1}{3}t^3$ =	8, 844504	0, 201248
$l\,N$ =	0, 935137	0, 935137

$l\,N\,t$ =

$l\,N\,t =$	0, 709012	1, 161260
$l\,\frac{1}{3}N\,t^3 =$	9, 779641	1, 136385
$N\,t =$	5, 11696	14, 4964
$\frac{1}{3}N\,t^3 =$	0, 60206	13, 6894
A Perihelio ad Elongationem maximam	5, 71902	28, 1858
	$5^d, 17^h, 15'$	$28^d, 4^h, 27'$
Perihelium erat Mart.	1, 8, 2	1, 8, 2
ſeu Febr.	30, 8, 2	
Febr.	24, 14, 47	29, 12, 29′ Mart.

Cometa ergo maximam habuit latitudinem borealem heliocentricam Menſe Februario 24^d, 14^h, $47'$, hinc ab Auguſto anni præteriti cometa usque ad hoc tempus ab ecliptica receſſit per tempus 202^d, 0^h, $35'$; inde autem ad eclipticam reverſus eſt 7 diebus 23 horis. Deinde poſtquam noſtris oculis ſe ſubduxiſſet ob nimiam declinationem auſtralem, latitudo ejus heliocentrica increvit usque ad Martii diem 29, 12^h, $29'$, quo tempore erat $47°$, $10'$, $53''$. Nunc igitur iterum ad eclipticam accedit, quam tamen non attinget niſi poſtquam ab aphelio reverſus pervenerit ad anomaliam veram $151°$, $25'$, $52''$, quod demum poſt complura ſecula eveniet.

39. Quanquam hanc poſtremam methodum adhibendo tres tantum cometæ obſervationes ſufficiunt ad ejus orbitam determinandam, tamen quoque in hunc finem plures obſervationes in uſum vocari, ſicque errores, qui forte in obſervationes irrepſerint inter omnes æqualiter diſtribui poterunt ut hoc modo orbita eo propius ad veritatem perducatur.

Sumta

ſumta ſcilicet quacunque obſervatione quarta, ex ea computetur tantum ejus elongatio a linea nodorum pro ſingulis tribus aſſumtis hypotheſibus neque enim ejus diſtantia a ſole opus erat. Quo facto ex elementis operum priorum obſervationum inventis, pro ſingulis quoque hypotheſibus quæratur tempus, quo cometa a perihelio ad eam anomaliam veram, quæ quartæ obſervationi competere deprehenſa eſt, pervenire deberet, ſeu computetur ad unamquamque hypotheſin, tempus, quod inter primam obſervationem & quartam interjacet, hocque intervallum cum obſervato comparatum, dabit novam æquationem pro erroribus x & y definiendis, ita ut omnino tres habeantur ejusmodi æquationes, ex quibus duplici modo has litteras x & y determinare licebit Quodſi utroque modo iidem pro his litteris valores prodeant, indicio id erit, jam primam determinationem eſſe exactam, ſin autem quæpiam diſcrepantia reſultet, tum pro x & y valores inter utrosque medii aſſumantur, quo aberratio theoriæ ab obſervationibus ſingulis circiter æqualis reddatur. Simili autem modo quinta, ſexta, pluresque obſervationes in ſubſidium vocari poterunt, quarum ope, ſi iſta præcepta obſerventur, vera cometæ orbita ſatis accurate definiri poterit, etiamſi ſingulæ obſervationes non nimis fuerint exactæ. Hac autem correctione orbita cometæ hic jam inventa non indigere videtur, cum omnibus obſervationibus tam prope ſatisfaciat, ut major conſenſus expectari non debeat.

Cometæ, qui Anno 1742 apparuit, loca obſervata ad tempus aſtronomicum medium ſtili veteris & meridianum Londinenſem reducta.

Locus obſervationis	Tempus æquale Londini St. vet.	Longitudo Cometæ	Latitudo Cometæ bor.
	Febr. 1742		
Peckini	$18^d, 8^h, 57'$	$9^s, 12^\circ, 24', 0''$	$16^\circ, 58', 0''$
Peckini	19, 10, 27	9, 13, 35, 0	22 , 54, 0
Peckini	20 , 8, 26	9, 14, 44, 0	28 , 4, 0
Peckini	21 , 9, 11	9, 16 , 2, 0	33 , 33, 0
Pariſiis	21, 17, 34	9, 16 , 6, 5	35 , 9 , 55
Pariſiis	22, 17, 34	9, 17, 46, 35	40, 58 , 37
Peckini	23 , 8, 26	9, 19, 32, 0	45 , 9 , 0
Petropoli	25, 10, 0	9, 24, 13, 0	56 , 29, 0
Peckini	27 , 6, 55	10, 3, 6, 0	66 , 22, 0
Petropoli	27, 10, 13	10, 3, 16, 0	66 , 41, 0
Petropoli	28 , 6, 49	10, 9, 8, 0	70 , 30, 0
Peckini	28 , 8, 55	10; 9, 56, 0	70 , 53, 0
Petropoli	28, 15, 8	10, 11, 56, 0	71 , 54, 0
	Mart.		
Peckini	1 , 7 , 39	10, 18, 19, 0	74 , 20, 0
Petropoli	1 , 7 , 57	10, 19, 12, 0	74 , 30, 0

Petropoli

Petropoli	1, 15, 10	10, 22, 13,0	75, 20, 0
Petropoli	2, 7, 2	11, 1, 49,0	77, 24, 0
Petropoli	2, 7, 39	11, 2, 15,0	77, 28, 0
Peckini	2, 8, 24	11, 2, 20,0	77, 33, 0
Pariſiis	2, 8, 44	11, 2, 30,5	77, 29, 3
Peckini	3, 7, 39	11, 19,20,0	79, 22, 0
Peckini	4, 8, 23	0, 8, 15,0	79, 59, 0
Petropoli	4, 14, 34	0, 13,20,0	79, 55, 0
Peckini	5, 8, 53	0, 26, 7,0	79, 31, 0
Peckini	6, 8, 23	1, 10, 11,0	78, 23, 0
Petropoli	6, 13, 55	1, 12, 24,0	77, 46, 0
Petropoli	6, 14, 15	1, 12, 44,0	77, 37, 0
Pariſiis	7, 7, 58	1, 19, 44,40	76, 42, 27½
Peckini	7, 8, 22	1, 20, 27,0	76, 49, 0
Londini	7, 10, 13	1, 20, 9,0	76, 38, 0
Pariſiis	7, 10, 37	1, 20, 34,0	76, 32, 10
Peckini	8, 0, 42	1, 24, 46,0	75, 53, 0
Peckini	11, 1, 21	2, 8, 19,0	71, 5, 0
Londini	11, 8, 33	2, 8, 32,0	71, 8, 0
Peckini	12, 2, 6	2, 10, 52,0	69, 41, 0
Peckini	13, 2, 36	2, 12, 53,0	68, 23, 0
Petropoli	15, 6, 45	2, 16, 18,0	65, 20, 0
Petropoli	16, 6, 38	2, 17, 31,0	64, 16, 0
Peckini	17, 0, 59	2, 18, 34,0	63, 32, 0
Peckini	17, [illegible], 49	2, 18, 56,0	63, 8, 0
Petropoli	17, 7, 50	2, 18, 52,0	63, 13, 0
Peckini	18, 6, 19	2, 19, 38,0	62, 21, 0

Peckini	19, 6, 43	2, 20,19,0	61, 33, 0
Peckini	20, 7, 8	2, 20,56,0	60, 47, 0.
Pariſiis	20, 9, 4	2, 21,12,30	60, 42, 40
Peckini	21, 8, 30	2, 22, 9,0	59, 43, [illegible]

Orbita hujus Cometæ determinata proſtat

Tom. VII. Miſcellan. Berolinenſium.

Diſtantia perihelii a ſole a = 73766 , 36

$l\ a$ = 4, 8678584

Semilatus rectum b eſt ad a ut 192952 ad 100000

Cometa per perihelium tranſiit Londini M. Jan. $21^d, 21^h, 52'$

Diſtantia perihelii a nodo aſcendente, $30° \ 32' \ 55''$

Longitudo nodi aſcendentis $6^s \ 16° \ 8' \ 55''$

Inclinatio orbitæ ad eclipticam $56° \ 35' \ 7''$

TABULA

TABULA MOTUS COMETAE IN PARABOLA.

Anom. vera $= v$ & $t =$ tang. $\frac{1}{2}$ v.

v. An. vera	$t + \frac{1}{3}t^3$ Area Parab.	$l(t + \frac{1}{3}t^3)$ Logar. Ar.	v. An. vera	$t + \frac{1}{3}t^3$ Area Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.
grad. 0	0, 0000000	$-\infty$	grad. 9.	0, 0788642	8, 8968799
	87271	∞		88477	461787
1.	0, 0087271	7,9408700	10	0, 0877119	8, 9430586
	87297	3010946		88747	418584
2.	0, 0174568	8,2419646	11.	0, 0965866	8, 9849170
	87351	1752024		89046	382992
3.	0, 0261919	8,4171670	12.	0, 1054912	9, 0232162
	87431	1260937		89374	353183
4.	0, 0349350	8,5432607	13.	0, 1144286	9, 0585345
	87536	971075		89731	327867
5.	0, 0436886	8,6403682	14.	0, 1234017	9, 0913212
	87672	794253		90114	306097
6.	0, 0524558	8,7197935	15.	0, 1324131	9, 1219309
	87831	67[illegible]339		90530	287215
7.	0, 0612389	8,[illegible]274	16.	0, 1414661	9, 1506524
	88019	583237		90976	270679
8.	0, 0700408	8,8453511	17.	0, 1505637	9, 1777203
	88234	515288		91451	255974
9.	0, 0788642	8,8968799	18.	0, 1597088	9, 2033177

v.

v. An. vera	$t + \frac{1}{2} t^3$ Ar. Parob.	$l(t + \frac{1}{3} t^3)$ Logar. Ar.	v. An. vera	$t + \frac{1}{3} t^3$ Ar. Parab.	$l(t + \frac{1}{3} t^3)$ Lo. Ar. Par.
grad. 18	0, 1597088	9,2033177	grad. 33	0, 3048770	9, 4841247
	91959	243240		103816	145422
19.	0, 1689047	9,2276417	34.	0, 3152586	9, 4986669
	92497	231548		104885	142137
20.	0, 1781544	9,2507965	35.	0, 3257471	9, 5128806
	93068	221149		106068	139059
21.	0, 1874612	9,2729114	36.	0, 3363539	9, 5267865
	93672	211763		107278	136352
22.	0, 1968284	9,2940877	37.	0, 3470817	9, 5404317
	94310	203259		108539	133731
23.	0, 2062594	9,3144136	38.	0, 3579356	9, 5538048
	94982	195525		109852	131283
24.	0, 2157576	9,3339661	39.	0, 3689208	9, 5669331
	95691	188466		111216	128990
25.	0, 2253267	9,3528127	40.	0, 3800424	6, 5798321
	96433	181997		112641	126850
26.	0, 2349700	6,3710124	41.	0, 3913065	9, 5925171
	97212	176060		114118	124843
27.	0, 2446912	9,3886184	42.	0, 4027183	9, 6050014
	98033	170599		115660	122971
28.	0, 2544945	9,4056783	43.	0, 4142843	9, 6172985
	98888	165557		117260	121216
29.	0, 2643833	9,4222340	44.	0, 4260103	9, 6294201
	99784	160895		118926	119577
30.	0, 2743617	9,4383235	45.	0, 4379029	9, 6413778
	100724	156582		120658	118045
31.	0, 2844341	9,4539817	46.	0, 4499687	9, 6531823
	101703	152575		122459	116614
32.	0, 2946044	9,4692392	47.	0, 4622146	9, 6648437
	102726	148855		124331	115276
33.	0, 3048770	9,4841247	48.	0, 4746477	9, 6763713

v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.	v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
grad. 48	0, 4744477	9, 6763713	grad. 62.	0, 6731708	9,8281252
	128281	114036		164373	104142
49.	0, 4872758	9, 6877749	63.	0, 6896081	9,8385394
	128303	112873		166905	103875
50.	0, 5001061	9, 6990622	64.	0, 7061986	9,8489269
	130409	111796		170584	103657
51.	0, 5131470	9, 7102418	65.	0, 7232570	9,8592926
	132601	110800		174422	103493
52.	0, 5264071	9, 7213218	66.	0, 7406992	9,8696419
	134876	109873		178421	103374
53.	0, 5398947	9, 7323091	67.	0, 7585413	9,8799793
	137244	109020		182590	103300
54.	0, 5536191	9, 7432111	68.	0, 7768003	9,8903093
	139707	108235		186942	103279
55.	0, 5675898	9, 7540346	69.	0, 7954945	9,9006372
	142270	107517		191480	103298
56.	0, 5818168	9, 7647863	70.	0, 8146425	9,9109670
	144935	106860		196220	103368
57.	0, 5963103	9, 7754723	71.	0, 8342645	9,9213038
	147708	106266		201166	103479
58.	0, 6110811	9, 7860989	72.	0, 8543811	9,9316517
	150594	105729		206334	103635
59.	0, 6261405	9, 7966718	73.	0, 8750145	9,9420152
	153598	105251		211731	103837
60.	0, 6415003	9, 8071969	74.	0, 8961876	9,9523989
	156725	104827		217378	104084
61.	0, 6571728	9, 8176796	75.	0, 9179254	9,9628073
	159980	104456		223277	104375
62.	0, 6731708	9, 8281252	76.	0, 9402531	9,9732448

$v.$ An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.	$v.$ An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
76.	0,9402531	9,9732448	91.	1, 3688601	10,1363592
	229451	104709		368111	115245
77.	0,9631982	9,9837157	92.	1, 4056712	10,1478837
	235911	105087		381665	116346
78.	0,9867893	9,9942244	93.	1, 4438377	10,1595183
	242677	105512		395965	117500
79.	1,0110570	10,0047756	94.	1, 4834342	10,1712683
	249760	105982		411070	118708
80.	1,0360330	10,0153738	95.	1, 5245412	10,1831391
	257185	106491		427036	119978
81.	1, 0617515	10,0260229	96.	1, 5672448	10,1951369
	264969	107052		443914	121302
82.	1, 0882484	10,0367281	97.	1, 6116362	10,2072671
	273131	107655		461777	122686
83.	1, 1155615	10,0474936	98.	1, 6578139	10,2195357
	281699	108304		480706	124140
84.	1, 1437314	10,0583240	99.	1, 7058845	10,2319497
	290689	109001		500758	125650
85.	1, 1728003	10,0692241	100.	1, 7559603	10,2445147
	300141	109749		522027	127129
86.	1, 2028154	10,0801990	101.	1, 8081630	0,2572376
	310074	110539		544617	128869
87.	1, 2338228	10,0912529	102.	1, 8626247	10,2701245
	320519	111378		568605	130604
88.	1, 2658747	10,1023907	103.	1, 9194852	10,2831849
	331508	112270		594116	132373
89.	1, 2990255	10,1136177	104.	1, 9788968	10,2964222
	343078	113209		621261	134256
90.	1, 3333333	10, 1249386	105.	2,0410229	10,3098478
	355268	114206		650185	136192
91.	1, 3688601	10, 1363592	106.	2, 1060414	10,3234670

Tabula I.

Fig: 1.

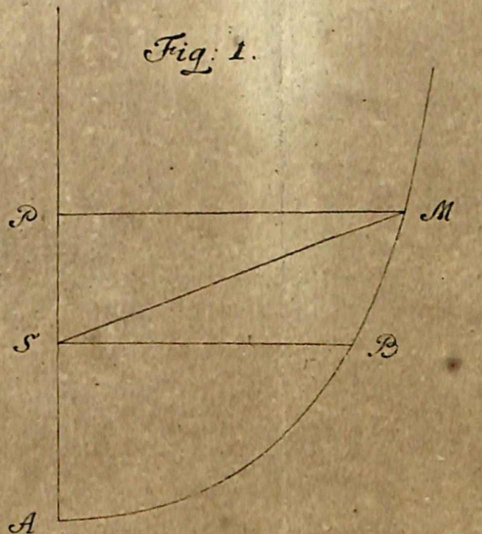


Fig: 2.

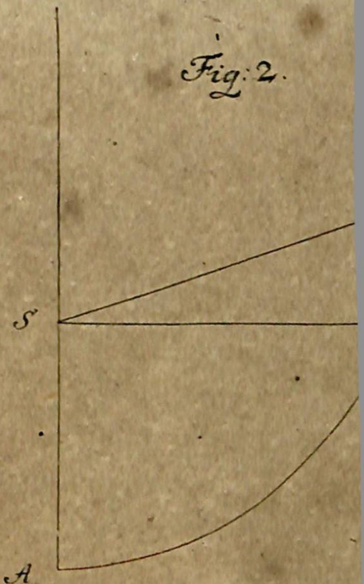


Fig: 3.

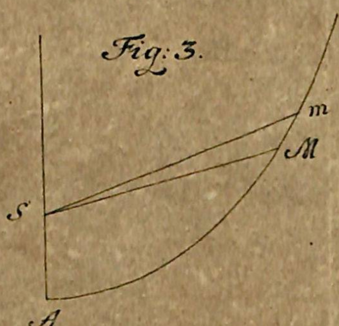


Fig: 4.

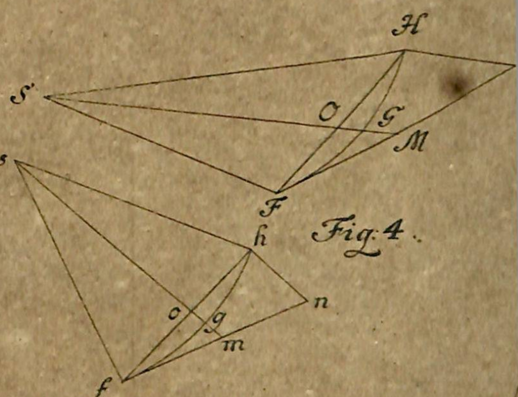


Fig: 6.

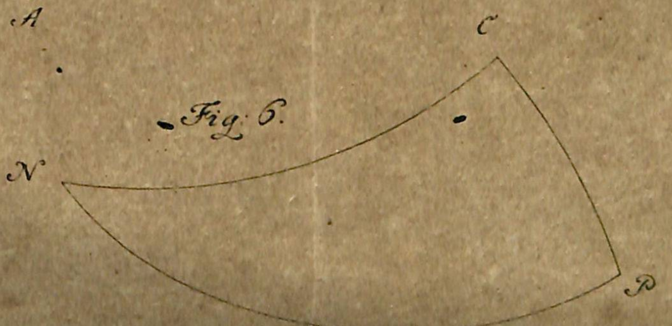


Fig: 8.



Tabula. II.

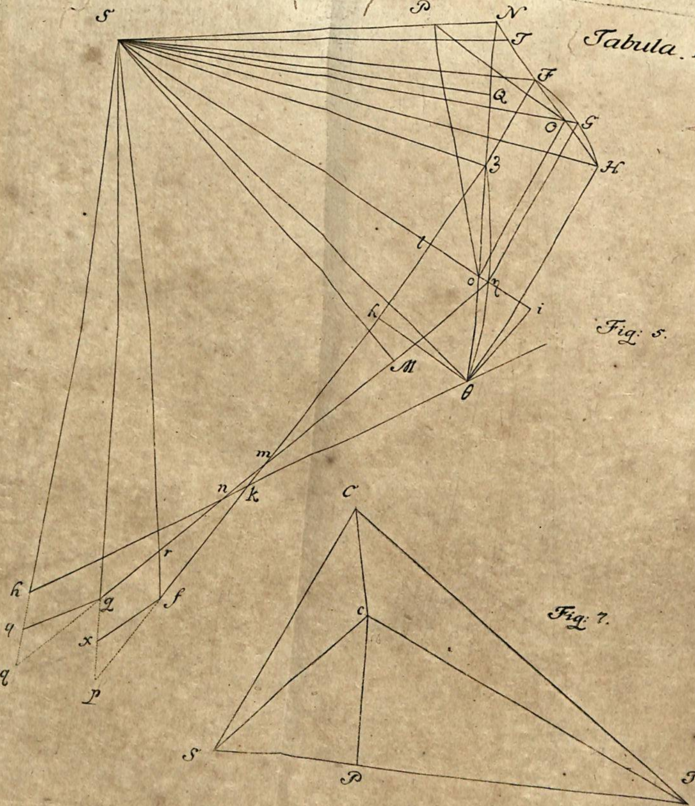


Fig: 5.

Fig: 7.

Tabula III.

Fig: 9.

Fig 11

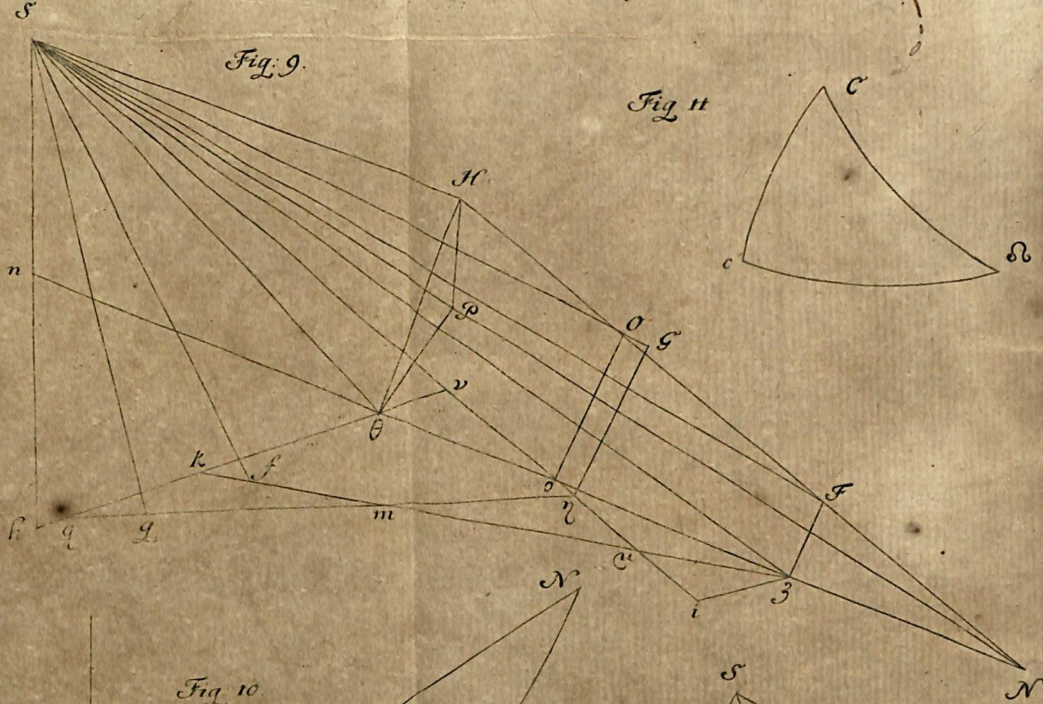
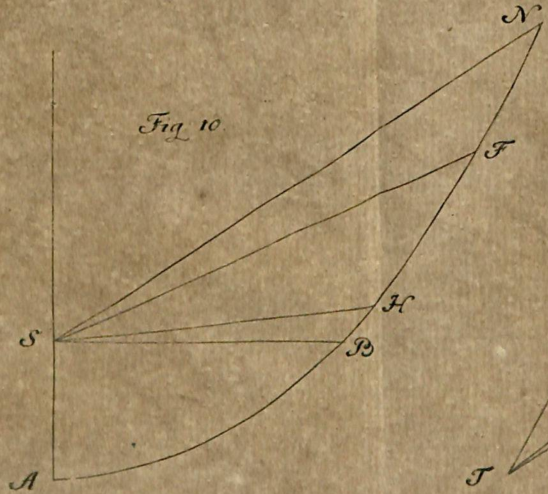


Fig 10.



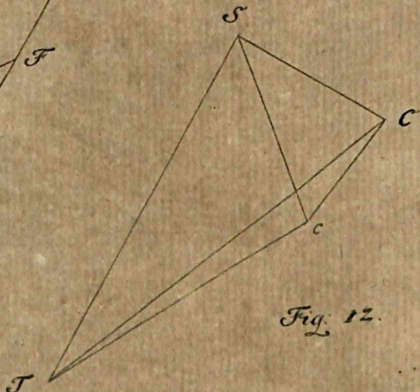


Fig: 12.

Tabula ad Additamentum

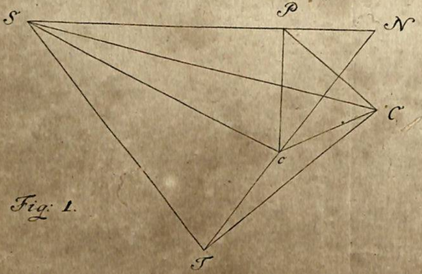
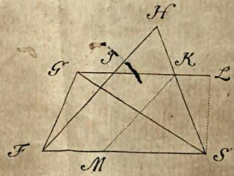


Fig: 1.

Fig: 3.



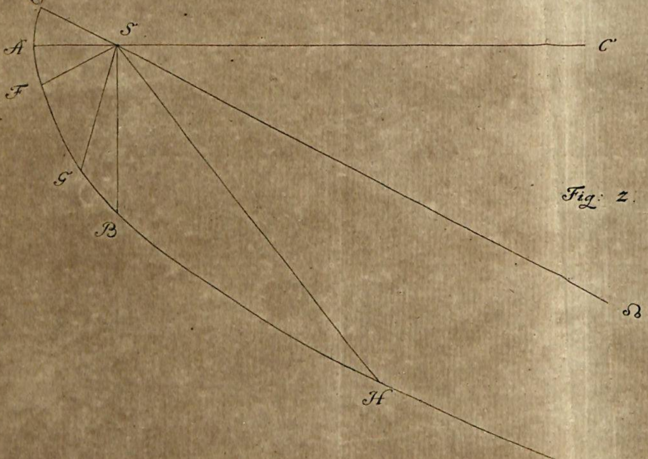


Fig: 2.

v. An.vera	$t + \frac{1}{2} t^3$ Ar. Parob.	$l\,(t + \frac{1}{3} t^3)$ Lo.Ar.Par.	*v.* An.vera	$t + \frac{1}{3} t^3$ Ar. Parab.	$l\,(t + \frac{1}{3} t^3)$ Lo. Ar. Par.
106.	2,1060414	10,3234670	120.	3, 4641016	10,5395906
	681009	138210		1439648	176840
107.	2,1741423	10,3372880	121.	3, 6080664	10,5572746
	715916	140702		1531254	180508
108.	2,2457339	10,3513582	122.	3, 7611918	10,5753254
	747051	142120		1630816	184339
109.	2,3204390	10,3655702	123.	3, 9242734	10,5937593
	786624	144783		1739121	188323
110.	2,3991014	10,3800485	124.	4, 0981855	10,6125916
	826836	147157		1857152	192478
111.	2,4817850	10,3947642	125.	4, 2839007	10,6318394
	869928	149623		1985965	196806
112.	2,5687778	10,4097265	126.	4, 4824972	10,6515200
	916132	152190		2126822	201322
113.	2,6603910	10,4249455	127.	4, 6951794	10,6716522
	965746	154857		2281094	206031
114.	2,7569656	10,4404312	128.	4, 9232888	10,6922553
	1019103	157641		2450375	210947
115.	2, 8588759	10,4561953	129.	5, 1683263	10,7133500
	1076486	160525		2636466	216076
116.	2,9665245	10,4722478	130.	5, 4319729	10,7349576
	1138350	163535		2841418	221434
117.	3,0803595	10,4886013	131.	5, 7161147	10,7571010
	1205088	166665		3067691	227035
118.	3,2008683	10,5052678	132.	6, 0228838	10,7798045
	1277168	169919		3317927	232890
119.	3, 3285851	10,5222597	133.	6, 3546765	10,8030935
	1355165	173309		3593349	238885
120.	3,4641016	10,5395906	134.	6, 7140114	10,8269820

$v.$ An.vera	$t + \frac{1}{3}t^3$ AreaParab.	$l(t + \frac{1}{3}t^3)$ Lo.Ar.Par.	$v.$ An.vera	$t + \frac{1}{3}t^3$ AreaParab.	$l(t + \frac{1}{3}t^3)$ Lo.Ar.Par.
134.	6,7140114	10,8269820	148.	17,625464	11,2461405
	3905592	245558		1,608797	379350
135.	7,1045706	10,8515378	149.	19,234261	11,2840755
	4246883	252144		1,824706	393616
136.	7,5292589	10,8767522	150.	21,058967	11,3234371
	4630260	259188		2,078751	408835
137.	7,9922849	10,9026710	151.	23,137718	11,3643206
	5059412	266572		2,379354	425102
138.	8,4982261	10,9293282	152.	25,517072	11,4068308
	5541169	274327		2,737148	442525
139.	9,0523430	10,9567609	153.	28,254220	11,4510833
	6083643	282480		3,165844	461238
140.	9,6607073	10,9850089	154.	31,420064	11,4972071
	669622	291052		3,683004	481379
141.	10,330329	11,0141141	155.	35,103068	11,5453450
	739010	300076		4,311613	503130
142.	11,069339	11,0441217	156.	39,414681	11,5956580
	817891	309590		5,081856	526683
143.	11,887230	11,0750807	157.	44,496537	11,6483263
	907868	319629		6,034015	552278
144.	12,795098	11,1070436	158.	50,530552	11,7035541
	1,010869	330233		7,222367	580197
145.	13,805967	11,1400669	159.	57,752919	11,7615738
	1,129269	341451		8,720981	610774
146.	14,935236	11,1742120	160.	66,473900	11,8226512
	1,265913	353338		10,632908	644416
147.	16,201149	11,2095458	161.	77,106808	11,8870928
	1,424315	365947		13,102930	681607
148.	17,625464	11,2461405	162.	90,209738	11,9552535

v.

v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
162.	90, 209738	11,9552535
	16, 339289	722960
163.	106,549027	12,0275495
	20, 64650	769223
164.	127, 19553	12,1044718
	26, 48042	821340
165.	153, 67595	12,1866058
	34, 54088	880526
166.	188, 21683	12,2746584
	45, 93237	948342
167.	234, 14920	12,3694926
	62, 45521	1026849
168.	296, 60441	12,4721775
	87, 15848	1118855
169.	383, 76289	12,5840630
	125, 43102	1228203
170.	509, 19391	12,7068833
	187, 30771	1360387
171.	696, 50162	12,8429220

v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
171.	696, 50162	12,8429220
	292, 67111	1523501
172.	989, 17273	12,9952721
	484, 0461	1729950
173	1473, 2188	13,1682671
	861, 6103	1999880
174.	2334, 8291	13,3682551
	1693, 0339	2368196
175	4027, 8630	13,6050747
	3802, 5759	2887114
176.	7830, 4389	13,8937861
	10737, 522	3749781
177.	18567, 960	14,2687642
	44167, 208	5287469
178.	62735, 168	14,7975111
	438917, 01	9028915
179.	501652, 17	15,7004026
180.	∞ infinitum.	∞ infinitum.

NOTA.

NOTA.

Orbita Cometæ A. 1742. quæ hic ex Volumine VII. Miſcellan. Berol. recenſetur, eſt ea quidem, quæ per correctionem ibi adhibitam prodiit, cum autem ea ſit nimis lubrica atque a vero vehementer ſeducere poſſit, ſi in obſervationes vel minimus error irrepſerit, præſtabit orbitam ibi primo inventam retinuiſſe, donec ea per methodum ultimam hic expoſitam emendetur: Erat ergo.

I. Diſtantia Perihelii a Sole = 75210

II. Semi-latus rectum = 150420

III. Cometa per perih. tranſiit Tempore Lond. medio St. vet. A. 1742. M. Januario $27^d, 4^h, 14'$

IV. Diſtantia Perihelii a nodo aſcendente $31^\circ, 17', 16''$

V. Longitudo helioc. Nodi aſcendentis $6^s, 9^\circ, 32', 7''$

VI. Inclinatio Orbitæ ad Eclipticam. 61, 43, 44

Berolini 1744.

Ære Michaelis.

00049837

Ex/ 23

Theoria Motuum Planetarum

CL/425 6/4/53 29/X

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